GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Spring Semester 2020

PLEASE WORK 3 OF THE 4 PROBLEMS ON THIS EXAM

DYNAMICS & VIBRATIONS

EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

✷ Please sign your **name** on the back of this page —

Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2020

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1

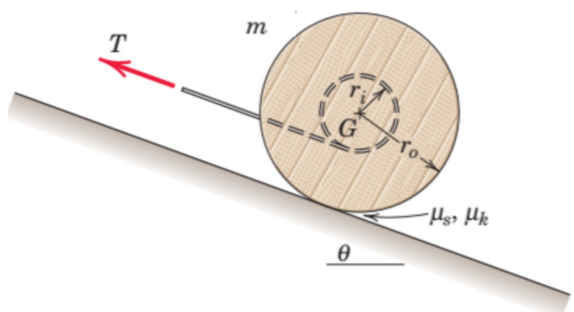
The spool of mass m and outer radius r_o can be approximated as a cylinder and has a massless central shaft of radius r_i. A tension T is applied to the end of a cable, which is wrapped securely around the central shaft as shown.

(a) The spool is observed to roll up the ramp without slipping. Find the angular acceleration of the spool. What is the condition such that rolling takes place?

(b) If the condition for rolling is later violated (e.g., the tension increases) at some time later, find the angular acceleration of the spool. Assume $T = 2mg$, $r_i = r_o/2$, and $\theta = 30°$.

(c) For part (a) would the angular acceleration increase or decrease in magnitude if the cable were wrapped around the shaft in the other direction so that it comes off the top side of the central shaft instead of the bottom? Why?

Note: Recall that the moment of inertia for a cylinder is $1/2mr^2$

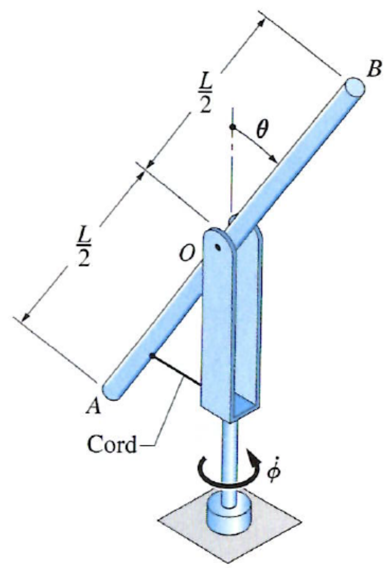


Problem 2

The uniform, slender rod *AB* has mass *m* and length *L*. It is pinned to a fork (at point *O*), with this fork attached to a vertical shaft. **The masses of the fork and shaft may be neglected, as well as any friction in the pinned joint.** The cord keeps the rod AB at a constant angle while the shaft is brought up to speed. Once the shaft sustains a constant angular speed, the cord is cut. **Show that the equations governing the ensuing motion are given by**

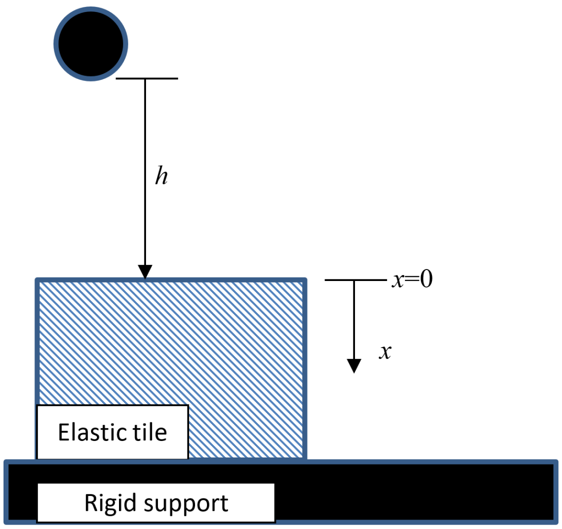
$$\ddot{\theta} = \dot{\phi}^2 sin(\theta)cos(\theta)$$

$$\ddot{\phi} = -2\dot{\theta}\dot{\phi}cot(\theta)$$



Problem 3

Many fitness centers have an elastomeric-tile flooring systems intended to manage the impact, or shock, due to the dropping of weights. Valuable insight on the performance of these systems may be obtained from simple single-degree-of-freedom analyses. Consider a mass m kg that is dropped from a height h m under the influence of gravity g m/s^2. It contacts the surface of a tile modeled as a single-degree-of freedom linear spring of equivalent stiffness k N/m; the spring is rigidly supported from below (this represents the actual structural floor, which is much, much stiffer than the tile). The mass causes the tile to deflect, bringing the mass to rest while converting kinetic energy into elastic strain energy; the mass then rebounds, and in the case of no losses, is launched off of the tile at the same velocity it had when it first contact the tile, but with opposite sign. This impact, from contact through to loss of contact, imparts a shock load, or pulse, into the floor.



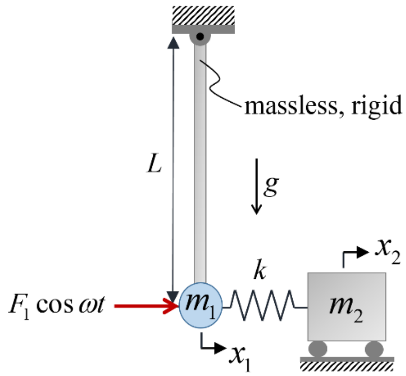
(a) Determine an expression for the maximum deflection of the tile, expressed in terms of g, h and the natural frequency of the mass m on spring k. What practical considerations might there be between this maximum deflection and the thickness of the tile?

(b) The total duration that the mass is in contact with the tile is the shock duration, that is, the length in time of the shock pulse delivered into the floor. Determine an expression for the duration of the shock pulse in terms of the natural frequency of the mass m on spring k. What is the implication of your expression for different drop heights?

(c) Determine the maximum force experienced by the floor during the shock in terms of m, g, h and the natural frequency of the mass m on spring k. What is the implication of your expression for different drop heights?

Problem 4

Consider the following undamped 2-DOF system for small oscillations of the pendulum.



(a) Express the forced equations of motion in matrix form.

(b) Consider the free vibration problem, i.e. $F_1 = 0$, along with the numerical parameters $m_1 = 10$kg, $m_2 = 5$kg, $k = 200$N/m, $L = 1$m, and $g = 9.81$m/s^2. Calculate the natural frequencies and mode shapes.

(c) Let $F_1 = 1$N and use the numerical parameters in part (b). Sketch the magnitude frequency responses $|x_1|$ vs. ω and $|x_2|$ vs. ω (show details, e.g. resonance and anti-resonance frequencies).

(d) In part (c), what is the excitation frequency at which the forced response $x_1(t) = 0$? What is the forced response $x_2(t)$ at this excitation frequency?

Note: "Forced response" is the so-called particular solution, i.e. you are expected to neglect the initial condition effects.