Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2019

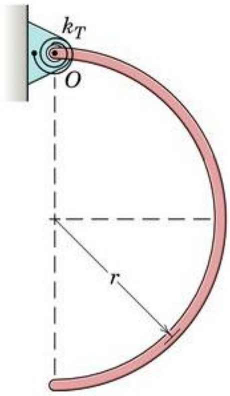
Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

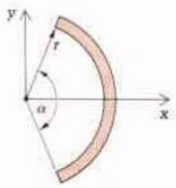
Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

The uniform portion of a semi-circular hoop of mass m is released from rest while in the position shown where the torsional spring of stiffness k_T has been twisted 90° clockwise from its undeformed position.

a) Determine the magnitude of the pin force at O at the instant of release. Motion takes place in a vertical plane and the hoop radius is r. Note that the center of the mass is a distance of $2r/\pi$ from the vertical centerline.
b) Would the magnitude of the horizontal and vertical components of the pin force increase or decrease if the hoop were a full-circle of the same mass?



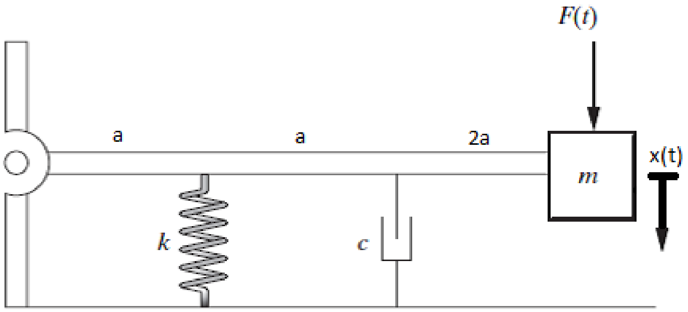
slender circular rod		$\left(\frac{r\sin(\alpha/2)}{\alpha/2}, 0, 0\right)$ $V = Ar\alpha$ (A = area of cross section)	$I_{xx} = \frac{mr^2}{2}\left(1 - \frac{\sin\alpha}{\alpha}\right)$ $I_{yy} = \frac{mr^2}{2}\left(1 + \frac{\sin\alpha}{\alpha}\right)$ $I_{zz} = mr^2$ (Note special cases for $\alpha = \pi$ and 2π.)

Problem 2

A foot pedal for a musical instrument is modeled by the sketch shown below. The tip displacement $x(t)$ is measured from the horizontal static equilibrium shown in the figure. With k = 1600 N/m, c = 25 kg/s, m= 25 kg, and $F(t)$ = 50cos2t N. Assuming small rotations about the pivot.

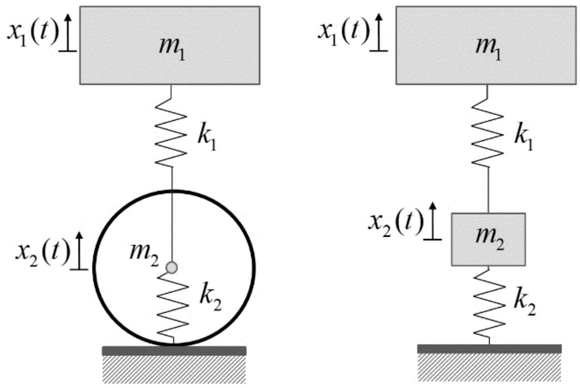
(a) Find the natural frequency and damping ratio of the system.

(b) Provide the steady-state response



Problem 3

Consider the following undamped quarter-car model and its 2-DOF representation, where the displacements are measured from the static equilibrium position. The masses m_1 and m_2 are the reduced vehicle mass and the lumped wheel-tire mass, respectively. The suspension spring of the quarter car is k_1, while the tire stiffness is k_2.



(a) For free vibrations of the system, derive the equations of motion and express them in matrix form.

(b) Calculate the natural frequencies and mode shapes for the following parameters: $m_1 = 250$ kg, $m_2 = 40$ kg, $k_1 = 20000$ N/m, and $k_2 = 100000$ N/m.

(c) At time $t = 0$, m_1 is given an initial displacement of 2 cm (all other initial conditions are zero), calculate the response for the same numerical parameters in part (b).

Problem 4

The uniform, slender bar *AB* weighs 20 lbs. It is connected to a vertical shaft *OC* by a ball-and-socket joint at *A*. The assembly rotates with a constant angular velocity ω. As a result, *OC* is inclined a constant 40 degrees to the vertical.

(a) Find the **reactions at *A*** and the **required angular velocity ω.**
(b) Would the reactions change in magnitude or direction if the assembly rotates in the opposite direction?