Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2016

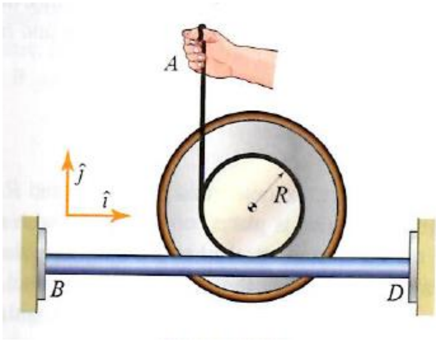
Instructions:

Please work 3 of the 4 problems on this exam. <u>It is important that you clearly mark which three problems you wish to have graded</u>. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

A cord is wrapped around the inner radius of a spool of mass *m* and radius of gyration k_G. The cord is pulled up by a constant force *P*, causing the spool to roll without slipping on bar *BD*. Assume the cord has negligible mass and is inextensible.



a) Determine the angular acceleration of the spool
b) Find the force vector between the spool and the bar *BD*

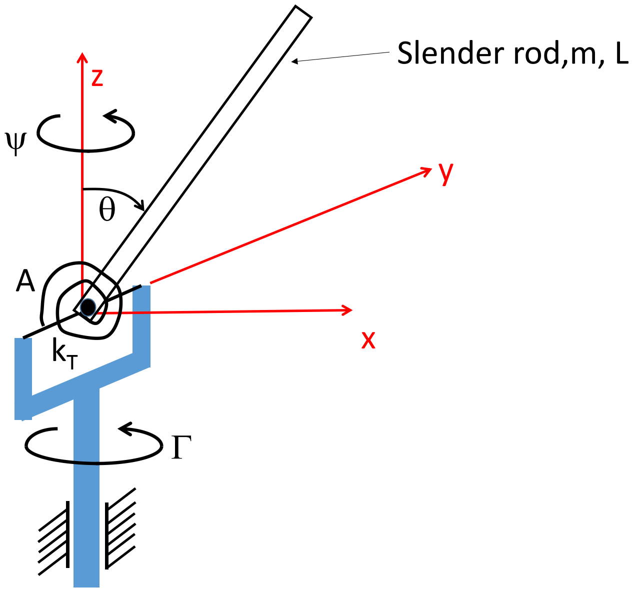
Problem 2.

A slender rod of mass m and length L is pinned at point A to a gimbal by means of a frictionless shaft as shown below. The xyz-axes are attached to a gimbal, which precesses about the z-axis with constant angular velocity $\dot{\psi}=\Omega$. The angle of inclination of the rod from the vertical is θ. A torsional spring k_T resists rotation of the rod about the y-axis, and is undeformed when $\theta = 0$. Additionally, gravity acts in the negative z-direction. Recall that the centroidal mass moment of inertia of a slender rod is $mL^2/12$.

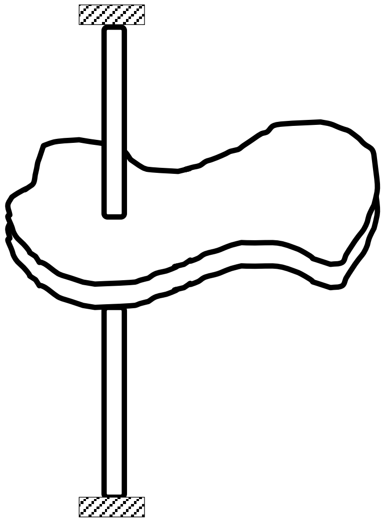
(a) Find a differential equation governing θ.

(b) Give an expression for the moment Γ necessary to maintain a constant precession rate of Ω.

(c) Linearize your result in (a) to obtain the condition for stable oscillations about $\theta = 0$.



Problem 3.



In modelling a high speed packaging machine it is necessary to know the inertia of an oddly shaped cam about its driveshaft and the stiffness of the driveshaft.

A co-worker vaguely remembers a method to do this by restraining the shaft in the vertical direction (see figure), observing its free vibration period, and then repeating with a known mass affixed to the cam. Note that the vertical rod is fixed at both ends and therefore provides torsional stiffness to the system. The cam lies in the horizontal plane.

Your job is to show how this can be done. Additionally, make a recommendation about affixing the mass to avoid potential numerical problems.

Problem 4.

Consider a 2-DOF vibratory system governed by

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{0\}$$

where $[M]=[M]^T$, $[C]=[C]^T$, and $[K]=[K]^T$ are real matrices.

(a) Assume that the system is proportionally damped such that

$$[C]=\alpha[M]+\beta[K]$$

where α and β are real constants. Express the modal damping ratios ζ_1 and ζ_2 in terms of α, β, ω_1, and ω_2 (where ω_1 and ω_2 are the corresponding undamped system's natural frequencies).

(b) Consider the undamped system ($[C]=[0]$) for the mass and stiffness matrices of

$$[M]=\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix},\ [K]=\begin{bmatrix} 2k & -k \\ -k & 3k \end{bmatrix}.$$

If one of the mode shapes is $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$, express the other mode shape <u>without</u> solving the eigenvalue problem.

(c) For the undamped system in part (b), solve the eigenvalue problem, obtain the natural frequencies and mode shapes.

(d) Let m = 1kg, k = 1N/m for the undamped system in part (b). Express the response to initial conditions $\{x(0)\}=\begin{Bmatrix} 20 \\ -10 \end{Bmatrix}\text{mm}$, $\{\dot{x}(0)\}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

<u>Hint</u>: By inspection, you may not need to do any derivation at all – why?

(e) Introduce proportional damping (of the form in part (a)) to the system in part (b). Let m = 1kg, k = 1N/m, $\alpha=0.1\text{s}^{-1}$, $\beta=0.1\text{s}$. Calculate the modal damping ratios ζ_1 and ζ_2.