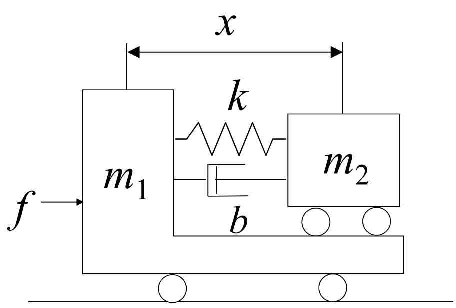
Work 3 out of 4 questions

Q1:

Problem: Consider the following mechanical system consisting of a spring, damper, and two masses. *f* is force and *x* is a relative displacement between mass 1 and mass 2.

(1) Obtain the transfer function from *f* to *x*.

(2) Use the transfer function obtained in (1). Assume $m_1=m_2=k=b=1$. Sketch the unit *impulse* response.



Q2:

It is generally recognized that *poles* and *zeros* of a dynamic system are influential in system performance. However, the influence is sometimes partially overlooked. We shall look into this.

Consider a dynamic system whose input-output transfer function is given by

$$\frac{Y(s)}{U(s)} = \frac{s+3}{s^2+3s+2}.$$

(1) Suppose λ is a *pole* of the system. Show that there are initial conditions of the system such that as $u(t) = 0,\ t \geq 0$, $y(t) = re^{\lambda t},\ t \geq 0$, where r is a constant.

Note that this result is interesting. There is no input to the system and yet the output keeps on coming out!

(2) Suppose λ is the *zero* of the system. Show that there are initial conditions such that as $u(t) = e^{\lambda t},\ t \geq 0$, we have $y(t) = 0,\ t \geq 0$.

Note that this result is interesting. One can keep adding "energy" to the system and yet nothing comes out!

In (1) and (2), you need to show all possible initial conditions and the explicit expression of r.

Q3:

A unity feedback system has the open-loop transfer function K/s(s+1)(s+4).

a) Sketch the root-locus.
b) What range of K gives a dominant 2^{nd} order overdamped response?
c) What range of K gives a dominant 2^{nd} order underdamped response?
d) Select a K so that the settling time is 16sec.
e) What range of K give a steady-state error less than 25% for a ramp input?

Q4:

The bottom graph shows the phase plot of KGH(s) where K=1 and the top graph is blank. Outside of the shown frequency range of 10^{-3} to 10^{3} the phase remains constant. Estimate the following:

a) KGH(s)
b) ω_{PC} - the phase crossover frequency
c) ω_{GC} - the gain crossover frequency
d) PM – the phase margin
e) db_{GM} – the gain margin
f) A new K so PM = 45°
g) db_{GM} in f)
h) With the new K in f) explain if the system is more or less stable and why.