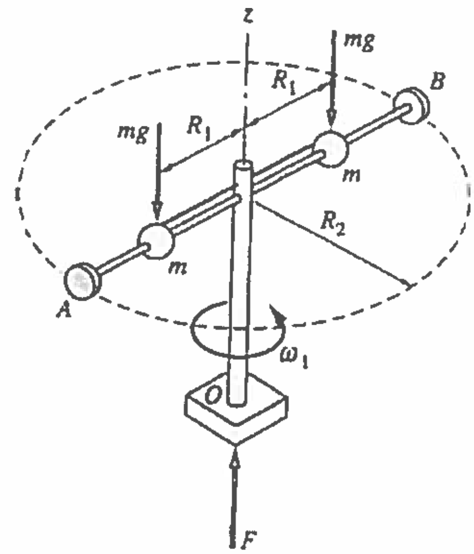
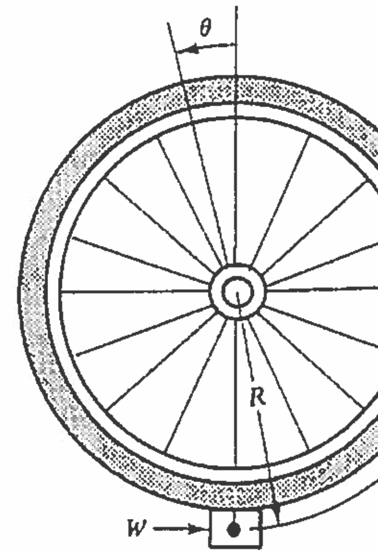
Problem 1

The assembly shown consists of two small balls, each of mass m, which slide on a frictionless, rigid frame OAB of negligible mass. The support at O permits free rotation of the frame about the z-axis. The frame is initially rotating with angular velocity ω_1 while strings hold the balls at a radial distance R_1. The strings are then cut simultaneously, permitting the balls to slide toward the stops at A and B. If the balls hit the stops and stay attached to the stops, determine the final angular velocity ω_2 of the assembly.



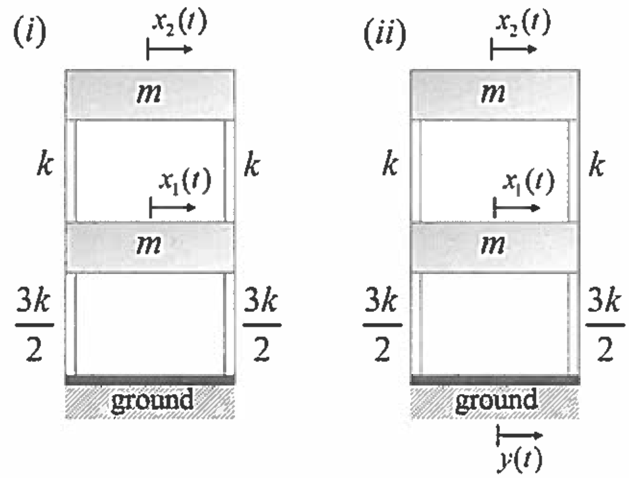
Problem 2

A bicycle wheel and tire are supported so that they are free to rotate about their centroidal axis through the hub of the wheel. A small weight W is glued to the tire, as shown in the figure, at a distance R from the axis of rotation. When the tire/wheel/weight assembly is displaced from the vertical by a small angle, the assembly is observed to oscillate 3 cycles every 10 seconds. If $R = 0.28$ m and $W = 3.34$ N, determine the centroidal mass moment of inertia I of the wheel and tire.



Problem 3

Consider the planar two-story building model shown below for (*i*) free and (*ii*) forced vibrations. Neglect damping and assume lumped-parameter behavior, i.e. assume rigid masses (m) and massless lateral stiffnesses (k). The small displacements $x_1(t)$ and $x_2(t)$ of the lumped masses are <u>measured relative to the ground in both cases</u>.



(a) Consider free vibrations and obtain the equations of motion in matrix form.

(b) Find the natural frequencies and mode shapes. Sketch the mode shapes.

(c) For the configuration in part (*i*), derive the response to initial displacement conditions $x_1(0) = 2\delta$, $x_2(0) = -\delta$, $\dot{x}_1(0) = \dot{x}_2(0) = 0$, where δ is a constant.

(d) For arbitrary ground excitation (displacement) $y(t)$ in part (*ii*), obtain the forced equations of motion in matrix form.

(e) For harmonic ground excitation $y(t) = Y_0 \sin \omega t$, find the forced response, i.e. particular solution (you do not have to use modal analysis). Neglect the initial condition effects.

<u>Note:</u> Your results must be in terms of the given constants and variables (m, k, $x_1(t)$, $x_2(t)$, etc.).

Problem 4

Consider two identical, homogeneous, railroad wheels (with small flanges) which spin about an adjoined axle with angular speeds σ_i (inner wheel i) and σ_o (outer wheel o) relative to the axle itself. Each wheel can be modeled as a cylinder with mass m and radius r. The axle has length l and can be considered light. The assembly is moving/turning as a whole on a horizontal, circular track with constant (inertial) angular velocity Ω. The circular track has a center O (not labeled) located a distance R from the first railroad wheel. The wheels roll without slipping on the track. For a sufficiently high Ω the inside wheel will lift off of the track. Find this value of Ω in terms of g, l, r, and R.