Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2013

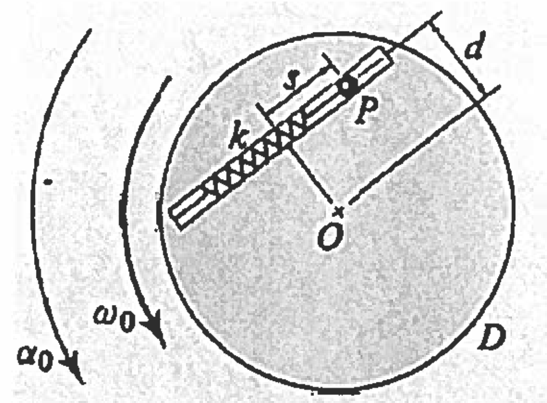
Instructions:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1

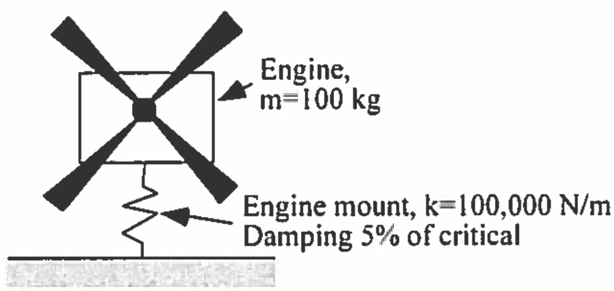
A small mass m slides in a slot relative to a rotating disk D. The slot is a distance d from the center of the disk. A linear spring of constant k is attached to the particle such that the spring is unstretched when the particle is at $s = 0$. The disk is rotating in a horizontal plane (i.e., no gravity) with a constant angular acceleration α_0. You can assume at time zero that the angular velocity of the disk is zero.



Determine the differential equation of motion governing s(t)

Problem 2

Consider a 4-bladed aviation propeller. Each blade has a mass of 15 kg, and may be modeled as a lumped mass at a distance of 1 m from the axis of rotation. The propeller spins at 2400 RPM. The overall system may be modeled as depicted below.



a) Determine the force amplitude on the engine due to the rotating imbalance that results when one of the blades falls off. Consider vertical motion, only.

b) For time long after the blade departs, such that all transients have died out, determine the steady-state amplitude of the engine on its engine mount.

Problem 3

Consider the schematic of an aircraft wing along with its 2-degree-of-freedom (DOF) model for the vertical displacements $x_A(t)$ and $x_B(t)$ of two turbofan engines. The fuselage end of the aircraft is taken to be the fixed end. The displacements of the engines are measured relative to this fixed boundary from the static equilibrium. Each engine has a total mass of m, the linear-elastic wing is massless, and the system is assumed to be undamped.

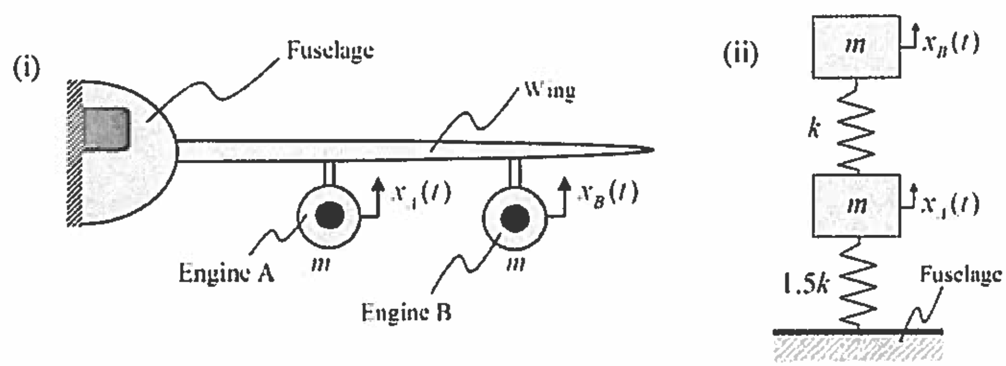


Figure (i) Front view of an aircraft wing and (ii) its 2-DOF model for bending vibrations.

(a) Derive the **equation of motion** for free vibrations of the 2-DOF system. Express the equation of motion in the **matrix form.**

(b) Find the **natural frequencies** and **mode shapes.** Sketch the mode shapes **of the wing.**

(c) Suppose that Engine A has an unbalanced mass of $0.001m$ rotating at $\omega = \sqrt{2k/m}$ with an eccentricity of e from the axis of its shaft. Find the **forced response amplitudes** of the two engines (forced response only, i.e. neglect the initial condition effects).

(d) What is the amplitude of the shear **force transmitted to the fuselage** due to the rotating unbalance in part c?

(e) What should be the nonzero rotation frequency ω (in part c) to make the force transmitted to fuselage (part d) equal to zero?

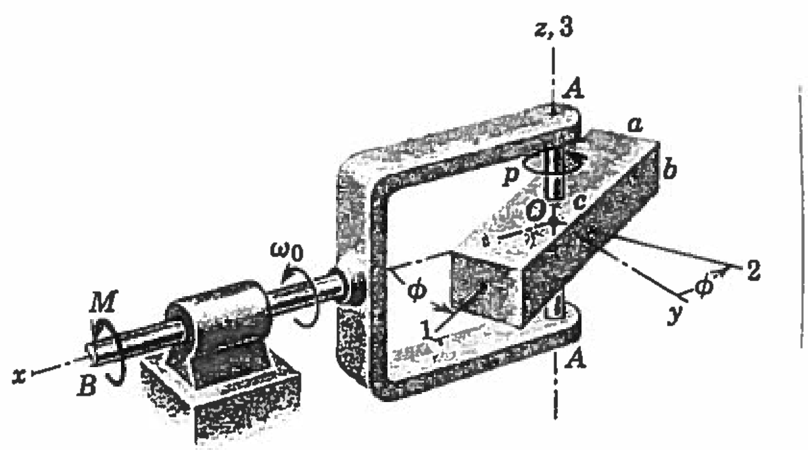
Note: Your results must be in terms of the given parameters: k, m, e.

Problem 4: GRADED

The homogeneous rectangular block of mass m is centrally mounted on the shaft A–A about which it rotates with a constant speed $\dot{\phi} = p$. Meanwhile the yoke is forced to rotate about the x-axis with a constant speed ω_0. Find the magnitude of the torque **M** as a function of ϕ. The center O of the block is the origin of the x-y-z coordinates. Principal axes 1-2-3 are attached to the block.

M as a function of

$\phi, p, \omega_0; m, c, a$



$$I_1 = \frac{m}{12}(a^2+b^2),\ I_2 = \dots,\ I_3 = \dots$$