Problem 1

Find $u(x,t)$ in the following problem:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}$$

where $-\infty < x < +\infty$ and $t \ge 0$. The initial condition is

$$u(x,0) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases} \quad \text{and} \quad u(0,0) = 1/2$$

Hint: Look for a solution in the form of $u(x,t) = \phi\left(\frac{x}{\sqrt{t}}\right)$

Problem 2

Consider a surface in 3D which is obtained by rotating the curve

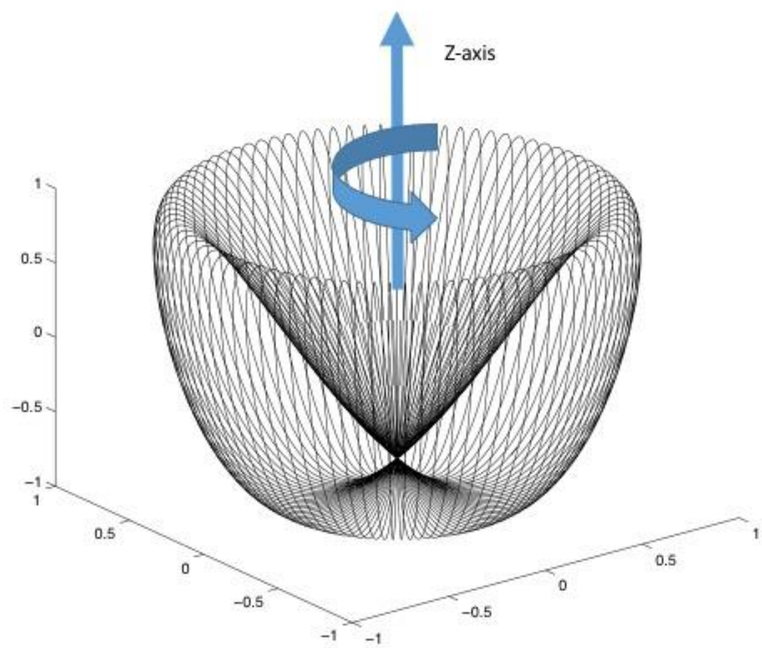
$r = cos\left(w - \frac{\pi}{8}\right)$ for $-\frac{\pi}{2} \le w \le \frac{\pi}{2}$

$z = sin(2w)$

around the z-axis. We assume that this volume is filled with matter. The density of the matter (density is mass per volume) is not constant but is a function of the coordinate along the z-axis as follows

$density = 5z^2$

The surface is depicted in the picture below.



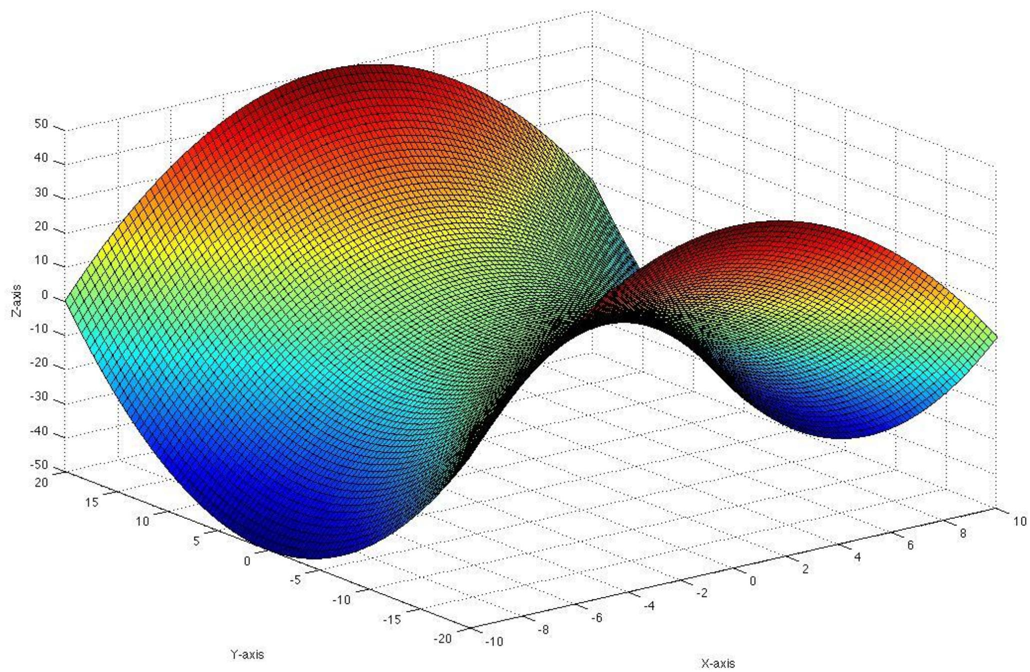

task : Use **Gauss' divergence theorem** to determine the total mass of the volume within the surface. Although the final resultis actually a real number you are only expected to solve the problem until your result contains no more than one integral in one variable.

Problem 3.

Assume a landscape, shown in the figure below, described by a hyperbolic paraboloid

$$x^2 - \frac{y^2}{4} = 2z$$

The surface separates soil and air, i.e. for each location air corresponds to larger z-values and soil corresponds to lower z-values. In the figure below therefore the air is above, the soil is below.




It can be verified that the location

$(x, y, z)$ =(-2,5,-9/8) is situated on the surface.

Now assume that you construct a post of length 30 perpendicular to the surface in the above mentioned location and pointing upwards into the air (not into the soil). At the top of the post an object is located that falls for an unknown reason. The object falls freely and vertically along a straight line parallel to the z-axis (because the gravitational force is directed parallel to the z-axis). Find the coordinates (x,y,z) where the object will hit the ground (i.e. where it will hit the surface).

Problem #4:

A function y=f(x) is defined by a set of {x,y} values at discrete points given in the table below.

| | | | | | |
|---|---|---|---|---|---|
| x | -3 | -1 | 0 | 1 | 3 |
| y | -4 | -0.8 | 1.6 | 2.3 | 1.5 |

**Part1:** If you are asked to find the least-square approximation of the function y=f(x) above using a polynomial of power "m" $P_m(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_m x^m$ develop:

(a) Expression for relevant error of approximation?

(b) Set of equations for finding the coefficients $a_0, a_1, \ldots a_m$ ?

**Part2**: Apply the methodology develop in Part 1b to find relevant coefficients $\boldsymbol{a_i}$ for two cases:

(a) Polynomial of power m=0

(b) Polynomial of power m=2

**Part3**: If you are asked to find least square approximation of the discrete-valued function y=f(x) defined at (n+1) points (e.g., see the table above) using $g(x) = a\cos(x) + bx^3$ , instead of a polynomial, develop a methodology for finding the unknown coefficient $\boldsymbol{a}$ & $\boldsymbol{b}$, which would produce the best fit to empirical data given by y=f(x)? Don't solve anything, just derive the relevant equations.