

Question 1

A is a 2x2 matrix given by

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

- 1. Is A invertible?**
- 2. Find the characteristic equation of A**
- 3. Find the eigenvalues of A**
- 4. Find the trace of A. How can the trace be used to verify that the eigenvalues are correctly calculated?**
- 5. Find the eigenvectors of A**
- 6. Express A as a triple factorization,
 $A = S\Lambda S^{-1}$
where the central matrix Λ is the diagonal matrix of eigenvalues.**
- 7. How could you calculate the the inverse of A without calculating its determinant?**

Question 3

- a. Find a unit vector normal to the plane $x - 2y + 2z + 5 = 0$.
- b. A triangle has vertices $(1,0,0)$, $(1,1,1)$, $(0,-2,3)$. Find its area.
- c. Determine the equation of the tangent plane at $(2,1,7)$ to the surface given by the equation $y = x^3 - yz$.

Question 4

Two functions in two variables are given.

$$f_1 = 2(x_1)^2 + (x_2)^2 - 8 = 0$$

$$f_2 = (x_1)^2 - (x_2)^2 + x_1 x_2 - 4 = 0$$

Question: find the value of x_1 and x_2 , for an acceptable error less than 0.2. Use the multi-variate Newton-Raphson method; begin your iteration with initial guess $x_1 = 1$ and $x_2 = 1$.