Applied Math Fall 2013

1.

a. Demonstrate explicitly that any function of the form *f(x+ct) – f(x-ct)* is a solution of the 1-D wave equation

$$\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

b. The 1-D vibration of a string of length L has a displacement *u(x,t)* governed by

$$\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

i. Pose boundary and initial conditions reflecting the fact that the ends of the string are held fixed and the string is released from rest and an initial state of displacement specified by a function *f(x)* that is consistent with the fixed ends.

ii. Solve the problem in i) by the method of separation of variables, considering **all** possible values for the separation constant.

2.

Consider an equation, called the Butler-Volmer equation, which in electrochemical processes relates density to overpotential. The significance of these technical terms need not concern us. For our purposes we extend the Butler-Volmer equation to obtain the following expression :

$$f(x) = -\beta + \exp(\alpha(x-2)) - \exp(-(1-\alpha)(x-3))$$

The values of α and β are complex :

$\alpha = 0.2\sqrt{-1}$

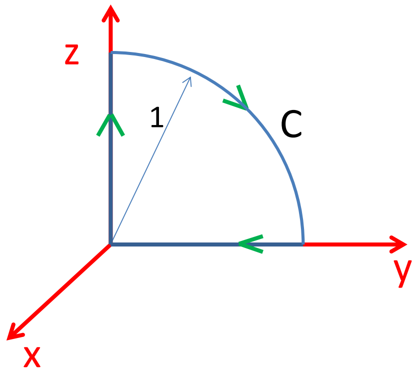
$\beta = -13 + 8.7\sqrt{-1}$

Limit your calculations to the interval $x \in [-1,3]$.

1) Give explicitly the analytical expression for Real(f(x))
2) Give explicitly the analytical expression for Imaginary(f(x))
3) Determine, numerically, the root of Imaginary(f(x)) <u>using the bisection method</u> (this must be clear in your answer!), with an error tolerance of 0.5 of the function value. You can use a calculator to calculate the different steps during your procedure.

3. Vector Calculus Problem

Consider the closed contour in the y-z plane and the vector field $\vec{F}(x,y,z) = y\vec{i} + z\vec{j} + x\vec{k}$. Compute the integral $\oint_C \vec{F} \cdot d\vec{s}$ two ways (a) Using Stokes' Theorem and (b) Directly



4.

Note that the parts of this question are unrelated. You can solve them independently.

a) Consider the subspace spanned by two vectors $\mathbf{a_1}$=[1,0,1,0,4] and $\mathbf{a_2}$=[2,0,0,0,4]. The matrix $\boldsymbol{P}$ projects every vector $\boldsymbol{b}$ to the nearest point in the subspace spanned by $\mathbf{a_1}$ and $\mathbf{a_2}$. What is the rank of $\boldsymbol{P}$ and *why*?

b) Find an orthonormal basis for the column space of $\boldsymbol{A}$ spanned by the vectors $\mathbf{a_1}$ and $\mathbf{a_2}$.

c) If $\boldsymbol{P}$ is any symmetric projection matrix, show that $\boldsymbol{Q = I - 2P}$ is an orthogonal matrix.