PhD Qualifying Examination in Acoustics

Two-hours, closed-book

Examinees are permitted a one page crib sheet

Answer all parts of all three questions

1) Consider a baffled circular piston that radiates in water. The radius of the piston is a=1 m. The expression for the pressure radiated in the far field by a baffled circular piston at distance r from the piston is

$$p(r,\theta,t)=\frac{j}{2}\rho_0 c U_0 \frac{a}{r} ka\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]e^{j(\omega t-kr)},$$

where J_1 is the Bessel function of the 1st kind, U_0 is the velocity of the piston.

a) (4 points) Assume that the frequency is 100 Hz. Using the given expression for the pressure in the far field, prove that the radiation resistance is approximately given by

$$R_r \approx \frac{1}{2}\rho_0 c S\left(ka\right)^2,$$

where S is the area of the piston.

b) (3 points) The frequency is increased to 100kHz. Find all angles at which the pressure amplitude in the far field is 0.

c) (3 points) Assume that for a frequency of 3 kHz, the sound pressure level on axis at 1 km from the piston is 100 dB re 1 μPa. Determine the rms speed of the piston.

(c) Zeros: Bessel Functions of the First Kind, $J_m(j_{mn}) = 0$

	j_{mn}					
m \ n	0	1	2	3	4	5
0	—	2.40	5.52	8.65	11.79	14.93
1	0	3.83	7.02	10.17	13.32	16.47
2	0	5.14	8.42	11.62	14.80	17.96
3	0	6.38	9.76	13.02	16.22	19.41
4	0	7.59	11.06	14.37	17.62	20.83
5	0	8.77	12.34	15.70	18.98	22.22

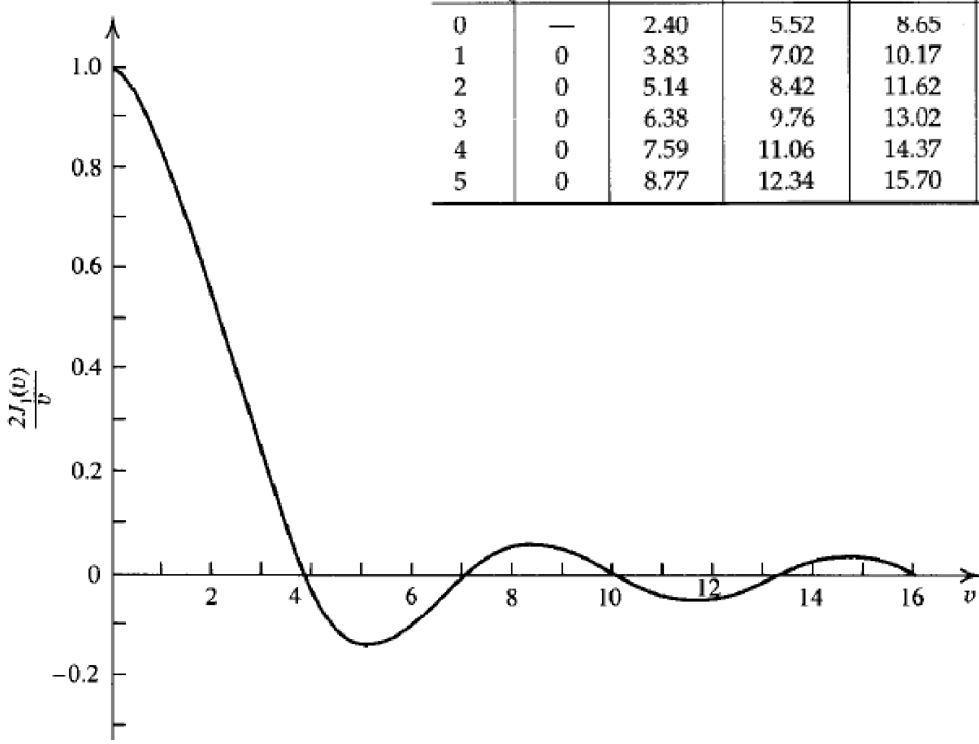


Figure 7.4.4 Functional behavior of $2J_1(v)/v$.

2) The Laplacian in cylindrical coordinates is given by

$$\nabla^2 p = \frac{\partial^2 p}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

a) Show that the standard wave equation in cylindrical coordinates (with no Θ and z dependence) reduces to

$$\frac{\partial^2(\sqrt{r}p)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2(\sqrt{r}p)}{\partial t^2} + \frac{\sqrt{r}p}{4r^2} = 0$$

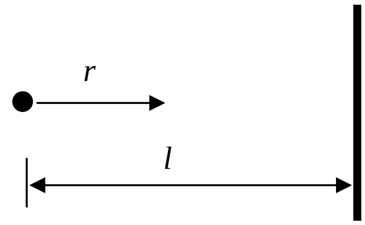
b) Show that for large values of the range variable *r*, the acoustic pressure is approximately given by

$$p = \frac{f(t - r/c)}{\sqrt{r}}$$

c) Find the reduction in sound pressure level corresponding to a doubling of distance, and contrast the result to the case of a spherical wave.

3) Consider a source a distance *l* away from a plane surface, as depicted below. The frequency-independent reflection coefficient for the plane is

$$R = |R| e^{j\varphi}$$



The source produces a sound field

$$p_s(r,t) = \frac{P}{r} e^{j(\omega t - kr)}$$

a) Using the method of images, determine the rms pressure squared as a function of distance *r* along the perpendicular between the source and the plane, with *r* measured from the source, and assuming that the source is emitting a pure tone of frequency f_c. Recall

$$p_{rms}^2 = \lim_{T\to\infty} \frac{1}{T} \int_0^T \left[p(t) \right]^2 dt$$

which reduces to a simple form when considering single harmonic signals.

b) Determine the rms pressure squared as above, but now assume that the source is emitting a broadband signal of bandwidth $B = f_2 - f_1$ centered on f_c, with fixed amplitude *P* independent of frequency. Note that the image source is NOT incoherent in this case, and that the band-total power is obtained by integrating the pure-tone result from part a) across the band *B*.

c) Discuss the significance of the results of parts *a* and *b*, with particular attention to pure tone and broadband measurements approaching a reflecting plane.