M.E. Ph.D. Qualifier Exam
Spring Semester 2002

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2002

Acoustics
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Acoustics Ph.D. Qualifying Examination

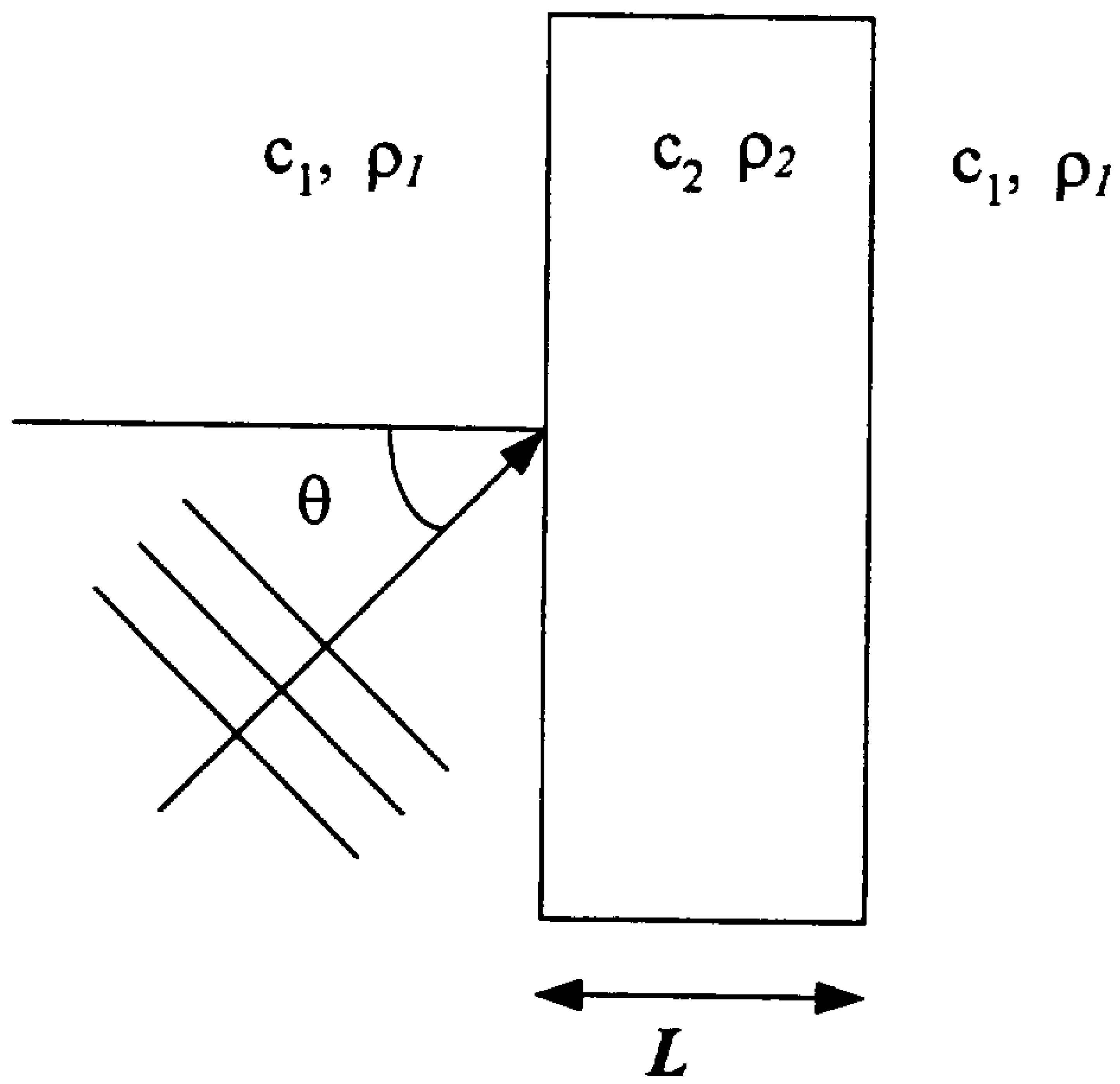
Closed book
Answer all questions

1. A time harmonic plane sound wave of angular frequency ω traveling in medium 1 (with speed of sound c_1 and density ρ_1) is incident on planar layer of medium 2 with thickness L (with speed of sound c_2 and density ρ_2) at an angle θ as shown in the figure.

   a) Formulate the problem to determine the pressure reflection coefficient $R(\omega,\theta)$. Show your reasoning and detailed steps of your formulation.

   b) Assuming normal incidence ($\theta = 0$) and that the embedded layer can be considered thin, obtain the leading order (non-vanishing) reflection coefficient for a thin layer.

   c) Considering the case where the acoustic impedance of medium 2 is very large as compared to that of medium 1, show how the density of medium 2 can be estimated from the measurement of reflection from a thin layer as in part b).

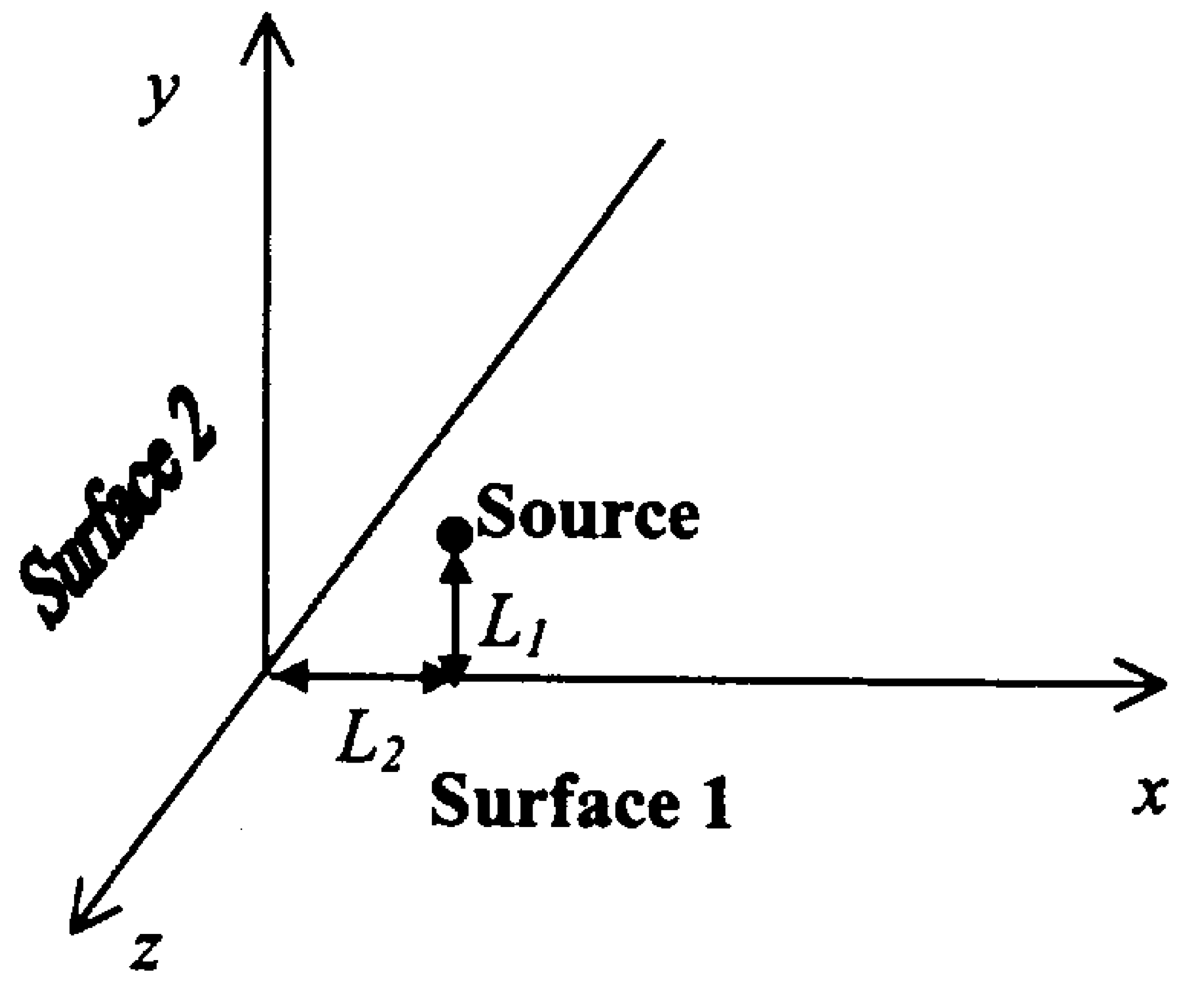


2. In free space a perfect point source produces an acoustic field given by:

$$p(R_d,t) = p_0 R_0 \frac{f(t-R_d/c)}{R_d}$$

where R_0 is some reference range , R_d is the distance from the source to the observation point. and f is some band-limited function.

Assume that two infinite surfaces meet at right angles, as shown below, so that the intersection forms the z axis of a Cartesian coordinate system. The source defined above is now located in the x-y plane at $(L_1, L_2, 0)$ a short distance, L_1, from surface 1 and a short distance, L_2, from surface 2.



Find an expression for the farfield pressure in terms of f, t, R_0 and spherical coordinates R,θ,ϕ (where $z = R\cos\theta$, $x = R\sin\theta\cos\phi$, $y = R\sin\theta\sin\phi$) $R \to \infty$ when

a) both surfaces are rigid (*i.e.*, zero normal velocity)

a. both surface are pressure release (*i.e.*, zero pressure)

Assume that L_1 and L_2 are much smaller than a wavelength at the highest frequency in the band.

3. **One-dimensional wave propagation in a lossless fluid in a gravitational field.**

Consider the one-dimensional problem of sound propagation along the z-axis, straight up in the atmosphere. Such a problem may be encountered in echo-sounders used for meteorology. Over long distances, the ambient properties of the atmospheres cannot be assumed to be independent of height z. Clearly the ambient pressure $P_0(z)$, the ambient density $\rho_0(z)$, and the ambient sound speed $c_0(z)$ all depend on height. The hydrostatic equilibrium requires that $\frac{\partial P_0}{\partial z} = -\rho_0 g$, where g is the gravitational constant.

a) Starting from the **exact** equations for continuity and momentum in a lossless fluid, in a gravitational field, linearize these equations.

b) The equation of state can be written as $\frac{Dp}{Dt} = c_0^2 \frac{D\rho}{Dt}$, where $p = P_0 + p'$ is the total pressure, $\rho = \rho_0 + \rho'$ is the total density, and where $\frac{D}{Dt} = \frac{\partial}{\partial t} + w\frac{\partial}{\partial z}$ is the material derivative, w being the particle velocity in the otherwise steady atmosphere. Show that the linearized equation of state becomes

$$\frac{\partial p'}{\partial t} + w\frac{\partial p_0}{\partial z} = c_0^2\left(\frac{\partial \rho'}{\partial t} + w\frac{\partial \rho_0}{\partial z}\right)$$

c) Combine the linearized equations of continuity, momentum, and state to derive the one-dimensional wave equation for the problem. The wave equation may be expressed in terms of the particle velocity, $w(z,t)$, as well as the static properties of the medium