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GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1999

Acoustics
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

1. A one dimensional acoustic field can be expressed as the sum of a travelling wave going to the right and a travelling wave going to the left. That is (using complex notation) we can always express the field in the following form

$$\hat{p}(x) = \hat{A}e^{ikx} + \hat{B}e^{-ikx}$$

a. Show that the field can also be expressed as the sum of two standing waves

$$\hat{p}(x) = \hat{A}_S \sin kx + \hat{B}_S \cos kx$$

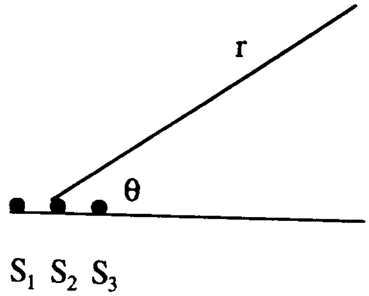
b. Express $\hat{A}_S$ and $\hat{B}_S$ in terms of $\hat{A}$ and $\hat{B}$.

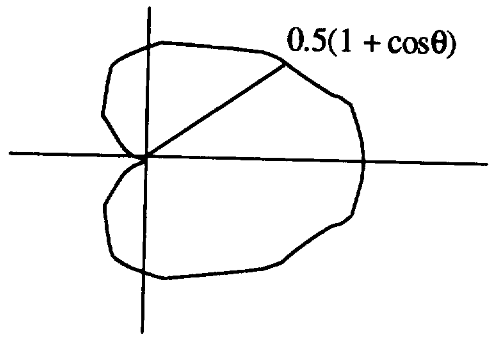
c. Show that the field can also be expressed as the sum of a standing wave and a travelling wave.

d. Is the decomposition of part c unique? If so, prove it. If not, give a counterexample.

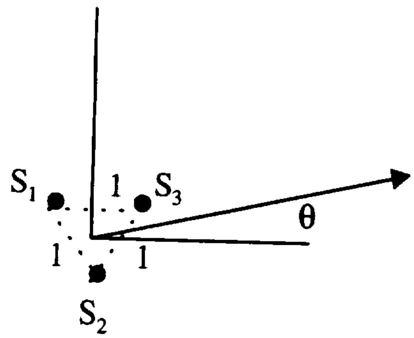
2. The figure below shows a configuration of three monopole sources, with strengths S_1, S_2, S_3.

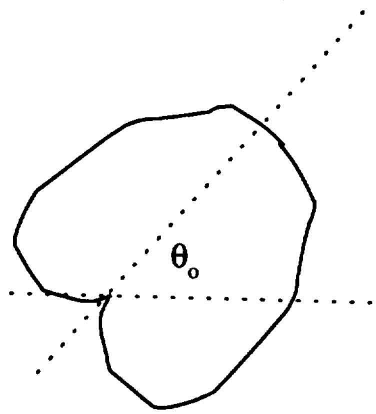
(a) Under what condition will this configuration produce the cardioid radiation pattern shown? Also, derive the expression for the time average total radiated acoustic power.





(b) Given the configuration shown below how could you choose the source strengths to produce a cardioid in the direction of θ_o, i.e. that has a radiation pattern given by $0.5(1 + \cos(\theta-\theta_o))$?





3. The mean speed of sound in a real gas as a function of temperature may be represented as

$$c(T) = \sqrt{\gamma R T}\,. \qquad (1)$$

We know that there is a temperature perturbation in an acoustic wave which is in phase with the pressure disturbance. Therefore, the speed of sound is actually different for different parts of a wave, depending on the local pressure. For a *finite* amplitude wave (pressure no longer small compared to ambient), this local speed variation may be significant. The temperature perturbation within a wave is

$$T' = \left(\frac{\partial T}{\partial p}\right)_0 p' \qquad (2)$$

where

$$\left(\frac{\partial T}{\partial p}\right)_0 = \frac{\gamma - 1}{\gamma}\frac{1}{R\rho} \qquad (3)$$

and where R is the gas constant, $R = R_0/M$, with R_0 the universal gas constant and M the molecular weight of the gas, and p' is the acoustic pressure.

a) Obtain an expression for the local speed of sound within a plane wave as a function of the mean speed of sound c_o, γ, and acoustic particle velocity u (Note: these should be the ONLY parameters in your result). Hint: you will need a Taylor series expansion of Eq. (1) about T_o, the plane wave impedance, and the relation $p=RT\rho$.

b) A more rigorous analysis than the one above yields the expression

$$c_{local} = c_o + u\frac{(\gamma + 1)}{2}$$

for the sound speed with respect to a fixed observer. Can you explain the difference between this expression and the one you derived above?

c) Comment on the physical significance of the results of parts A and B, particularly as it relates to long-range propagation. Assuming the temperature effect considered above is the only factor different from your prior knowledge of linear acoustics, does a finite amplitude wave's shape remain constant as it propagates?

M.E. Ph.D. Qualifier Exam
Spring Quarter 1999

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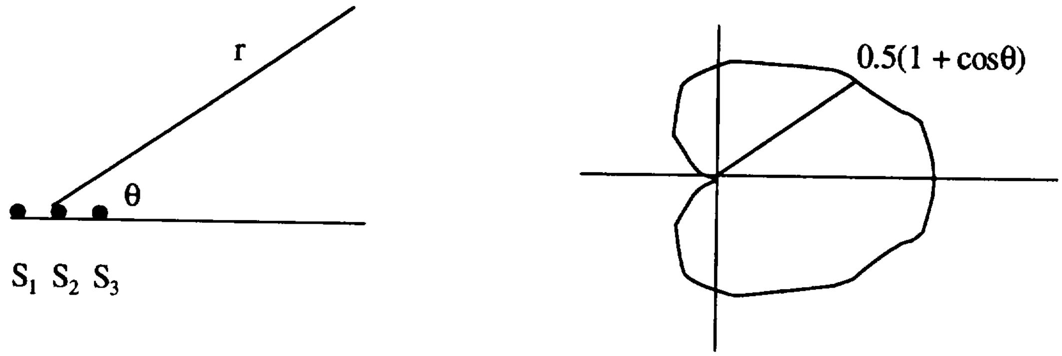

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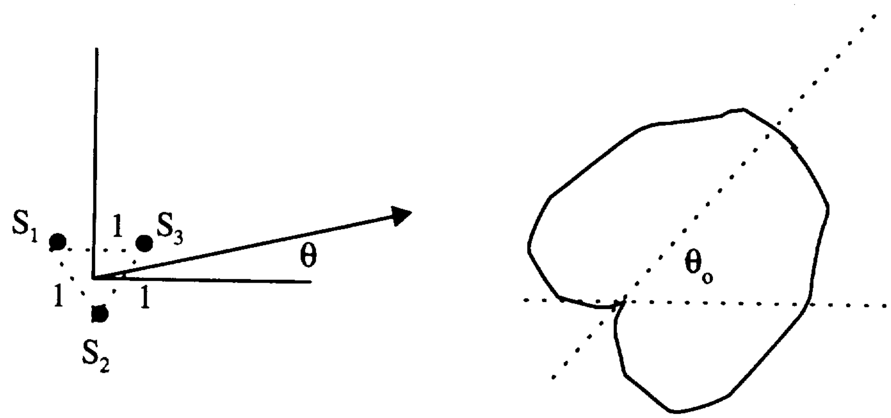

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