## PhD Qualifying Examination in Acoustics

## Two-hours, closed-book . Answer all parts of all three questions.

1. Consider a set of stairs. If an impulsive sound (such as a handclap) is produced at a point $\boldsymbol{R}$ in front of the stairs as depicted in the picture below, a "chirp" echo is produced. The chirp is of finite length, and of varying frequency content with time. The sound has been likened to that of the Quetzalcoatl bird. If there are $N$ stairs in the set, each with a rise of $L$ and tread depth $T$ :
a) Determine the approximate duration of the echo.
b) Sketch the time history of the echo, and label its significant features
c) Determine whether the dominant frequency of the echo increases or decreases with time, and explain your reasoning.
d) Estimate the dominant frequency at the start of the echo.
e) Estimate the dominant frequency at the end of the echo (you may assume that $N$ is large enough such that the incident wave is at grazing incidence, that is, it's parallel to the slop of the stair).
2. For plane wave reflection from a fluid-fluid interface it is observed that at normal incidence the pressure amplitude of the reflected wave is one-half that of the incident wave (no phase information is known). As the incidence angle is increased, the amplitude of the reflected wave first decreases to 0 and then increases until it reaches an amplitude of 1 for an incidence angle of 30 degrees
3. Prove that the expression for the pressure reflection coefficient for oblique incidence is
$R=\frac{\frac{r_{2}}{r_{i}}-\cos \theta_{v} / \cos \theta_{t}}{\frac{r_{2}}{r_{1}}+\cos \theta_{t} / \cos \theta_{i}}$
Where $\boldsymbol{\theta}_{\boldsymbol{t}}$ is the angle of transmission, $\mathrm{r}_{1}$ is the characteristic impedance of fluid $1, \mathrm{r}_{2}$ is the characteristic impedance of fluid 2. (3 points)
4. Find the speed of sound and density of medium 2 if medium 1 is water. (4 points)
5. Derive an equation for the incident angle that maximizes power transmission.

Express this incident angle as a function of the properties of medium 1 and medium
2. (3 points)

Bold font denotes complex variables in the problem statement. All three questions can be solved independently.
3. Recall that the radiated pressure amplitude by a harmonic monopole of strength $Q$ at a distance $r$ is given by

$$
p(r)=-i \omega \rho_{o} Q \frac{e^{i k r}}{4 \pi r}
$$



1) (5pts) Show that the radiated pressure amplitude $P_{d}$ in the far-field by two monopoles of respective source strength ( $+Q_{1}$ and $Q_{2}$ ), located at a distance $d$ apart is given by

$$
p_{d}=\left(1+\frac{Q_{2}}{Q_{1}} e^{-i k d \sin \vartheta_{N}}\right) p_{1}
$$


where $p_{1}$ is the radiated pressure by the first monopole (here the one on the left). i.e.

$$
p_{1}=i \omega \rho_{o} Q_{1} \frac{e^{i k r}}{4 \pi r} .
$$

2) (2pts) Simplify this expression assuming that $k d \ll 1$, and $Q_{2}=-Q_{1}$.
3) (1pt) Sketch the beam-pattern of the radiator obtained in part 2.
4) (2pts) Considering your result for part 2; what kind of radiator is it? Comment on its apparent source strength with respect to the source strength of the first monopole.
