GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1996

ACOUSTICS
____
EXAM AREA

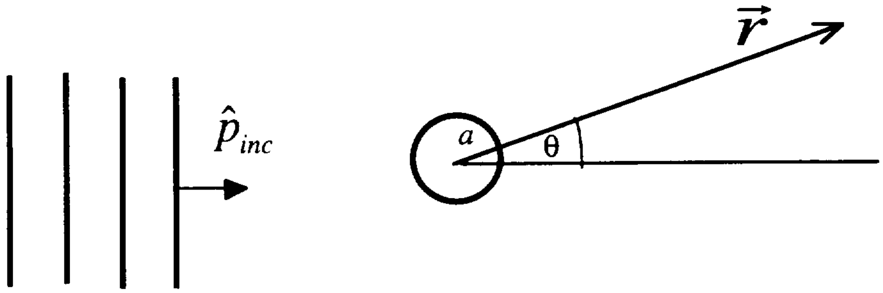
____

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --

Acoustics Qualifying Exam, Fall, 1996

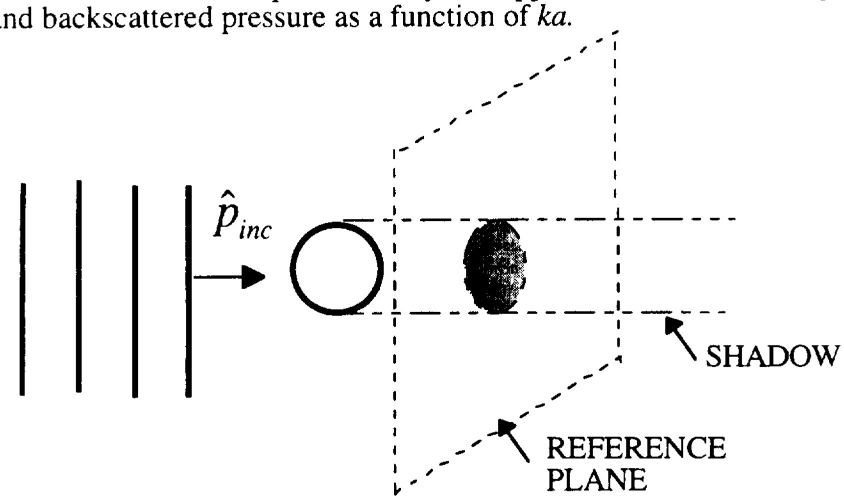
Work all 4 problems. Show all of your work. Clearly state all assumptions.



A plane harmonic wave $\hat{p}_{inc}$ with wavenumber k is incident on a pressure release ($\hat{p}_{total} = 0$ boundary condition) sphere of radius a . The *scattered* pressure is defined as the difference between the total pressure and the incident pressure. That is

$$\hat{p}_{scat} = \hat{p}_{total} - \hat{p}_{inc}$$

a. Assuming that the sphere is acoustically small (*ka<<1*) find expressions for the forward scattered($\theta = 0$) and back scattered ($\theta = 180^o$) **farfield** pressure by approximating the **scattered** field as the sum of the fields of a point monopole and a point dipole located at the center of the sphere. Justify this approach. Sketch the magnitude of the forward and backscattered pressure as a function of *ka.*



b. At very high frequencies (*ka>>1*) the sphere casts a shadow (with circular cross-section) in the nearfield behind the source. Assuming that the shadow is very sharply defined, estimate the farfield forward scattered pressure by considering the radiation from a plane located on the far side of the source. (See above figure) How does the magnitude of the forward scatter at low *ka* compare with its magnitude at high *ka*?

Structural acoustic response of finite rib-reinforced plates

Richard F. Keltie

Center for Sound and Vibration, Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, North Carolina 27695-7910

(Received 4 May 1992; accepted for publication 23 April 1993)

The structural acoustic response of finite fluid-loaded flat plates with arbitrary attached rib stiffeners was formulated. The ribs were modeled as simple inertial reactions providing transverse forces to the plate structure. The plate response and the surface acoustic pressure distribution were written as expansions in terms of the *in vacuo* plate eigenfunctions. The expansion coefficients were obtained by solving a set of simultaneous complex linear algebraic equations. Results were obtained for an unribbed steel plate, and for a steel plate with a set of uniform ribs, in response to harmonic line-force excitation. The results demonstrated the potential for significant reductions in the vibration level by approximately 90% at certain frequencies because of the presence of the attached ribs. These reductions were attributed to the interaction of two nearly resonant modes in a "beating" phenomenon. It was shown that this "beating" arises because of the presence of the ribs. In order to quantify this effect, a quantity called the "rib function" was defined and was shown to be dependent upon the rib masses, their locations, and the *in vacuo* mode shapes.

PACS numbers: 43.40.Dx

INTRODUCTION

The vibration and acoustic response of structures with attached stiffeners or masses has been the subject of numerous studies, which may be grouped into a few categories: (a) periodic systems of infinite extent;[1–8] (b) infinite systems with a finite number of attached stiffeners;[9–13] and (c) infinite systems with slightly disordered nearly periodic attached systems.[14–17] In (a), the classical results involving passbands and stop bands were obtained. These results indicate that in periodic systems, the reflected and transmitted structural waves interact in such a manner as to allow certain frequencies to pass without attenuation, while other frequencies are attenuated dramatically. This class of problem also produced the concept of propagation constants in structural wave transmission. Recent work involving the addition of fluid loading in this class of problem indicates that the fluid path allows for energy propagation along the structure, mitigating the strong stop-band effects.[18]

In (b), emphasis was placed on characterizing the effects of one, or a few, attached stiffeners. The primary item of interest was the degree to which an incident structural wave was attenuated by the attached system, and how small numbers of attached systems could interact. Category (c) consists of relatively recent developments in the application of what is called "Anderson localization" to structural vibration. This phenomenon may be described by the strong localization of the resulting vibration in the neighborhood of the excitation. The underlying physics involves the slight phase shifts of the reflected waves due to the irregularity in the stiffeners' locations. These phase shifts disrupt the coherence necessary to propagate energy over substantial distances.

In comparison to the three categories just discussed, there has been relatively little activity dealing with finite systems with attached ribs. While the wave approach has been favored for its ease of manipulation and interpretation in dealing with infinite systems, finite systems lend themselves to a modal description. Such a description is often less amenable to interpretation for physical understanding, but it is representative of practical engineering systems and structures. The results presented here are an attempt to address the finite system case and to develop physical understanding of the effects of ribs on the associated structural response.

I. FORMULATION

The system considered is shown schematically in Fig. 1. Consider a uniform plate lying in the x-y plane with an acoustic medium above the plate, $z>0$, and a vacuum below, $z<0$. The plate has finite extent in the x direction given by L and is of infinite extent in the y direction. The supports of the plate are assumed to be described by simple supports. The flexural stiffness of the plate is denoted by D and the mass/unit area of the plate is denoted by ρh. If the plate is excited by a line force of amplitude/unit length F_0 and circular frequency ω acting along $x=x_0$, then the equation of motion in terms of the plate displacement $w(x)$ is given by

$$D\frac{d^4w(x)}{dx^4}-\rho h\omega^2 w(x)=F_0\delta(x-x_0)-p_a(x,0)-\sum_{r=1}^{R}p_r(x), \qquad (1)$$

where $p_a(x,0)$ is the acoustic pressure acting on the plate's surface and the term $p_r(x)$ represents the reaction pressure from each of the R attached ribs. The ribs are considered to react only in a normal direction, so that any moment reaction or in-plane reactions are neglected. Furthermore, because of the presence of the line force there will be no

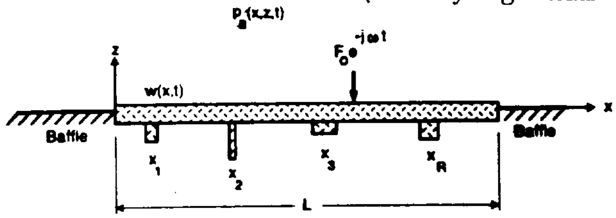


FIG. 1. Schematic diagram of problem geometry.

flexure of the ribs, so that they provide purely inertial reactions. Then the $p_r(x)$ are given by

$$p_r(x)=-m_r'\omega^2 w(x)\delta(x-x_r), \quad (2)$$

where m_r' and x_r represent the mass per unit length and the location of the rth ribs, respectively. This elementary rib model is used in order to expedite the development of a model that incorporates completely arbitrary rib sizes and locations.

The acoustic pressure above the plate, $p_a(x,z)$, satisfies the Helmholtz equation

$$\frac{\partial^2 p_a}{\partial x^2}+\frac{\partial^2 p_a}{\partial z^2}+k_0^2 p_a=0, \quad (3)$$

where k_0 is the acoustic wave number (ω/c_0). The boundary condition on the equality of the normal velocity of the plate and the normal particle velocity in the fluid is given by

$$\left.\frac{\partial p_a}{\partial z}\right|_{z=0}=-\rho_0\omega^2 w(x), \quad (4)$$

where ρ_0 is the ambient acoustic fluid density.

The plate displacement may be written as an expansion using the *in vacuo* vibration modes,[19] so that

$$w(x)=\sum_{n=1}^{N} W_n\phi_n(x), \quad (5)$$

where the $\phi_n(x)$ functions are the normalized plate eigenfunctions:

$$\phi_n(x)=\sqrt{2/\rho h L}\,\sin(n\pi x/L). \quad (6)$$

The acoustic pressure on the surface $z=0$ is written as

$$p_a(x,0)=\sum_{n=1}^{N} P_n\phi_n(x). \quad (7)$$

The series for the plate displacement and the series for the acoustic pressure are now substituted into Eq. (1). The resulting equation is then multiplied by an arbitrary normalized mode shape, say, $\phi_s(x)$, and integrated over the extent of the plate. Because of the orthogonality of the modal functions, the resulting equation simplifies to

$$(\omega_s^2-\omega^2)W_s=F_0\phi_s(x_0)-\frac{P_s}{\rho h}+\sum_{r=1}^{R} m_r'\omega^2\phi_s(x_r)\sum_{n=1}^{N} W_n\phi_n(x_r), \quad (8)$$

where ω_s is the *in vacuo* natural frequency of the sth plate mode.

In order to determine the pressure expansion coefficients P_s in terms of the displacement expansion coefficients W_s, the Fourier transforms of acoustic pressure and the plate displacements are introduced:

$$\bar{P}_a(\alpha,z)=\int_{-\infty}^{\infty} p_a(x,z)e^{-j\alpha x}\,dx \quad (9)$$

and

$$\bar{W}(\alpha)=\int_{-\infty}^{\infty} w(x)e^{-j\alpha x}\,dx. \quad (10)$$

By utilizing the boundary condition [Eq. (4)], the surface acoustic pressure may be given by the inverse Fourier transform:

$$p_a(x,0)=\frac{-j\rho_0\omega^2}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{W}(\alpha)}{\sqrt{k_0^2-\alpha^2}}e^{j\alpha x}\,d\alpha, \quad (11)$$

where the correct branch of the radical is defined by

$$\sqrt{k_0^2-\alpha^2}=\begin{cases}\sqrt{k_0^2-\alpha^2}, & k_0>|\alpha|,\\ j\sqrt{\alpha^2-k_0^2}, & |\alpha|>k_0.\end{cases} \quad (12)$$

The series expansion for the surface acoustic pressure and the inverse transform of the plate displacement are now substituted into Eq. (11). The resulting equation is then multiplied by $\phi_s(x)$ and integrated over the plate's surface. The relationship between the pressure expansion coefficients and the displacement expansion coefficients is then given by

$$P_s=\frac{-j\rho_0\omega^2\rho h}{2\pi}\sum_{n=1}^{N} W_n Z_{ns}, \quad (13)$$

where Z_{ns} is defined as

$$Z_{ns}=\int_{-\infty}^{\infty}\frac{\Phi_n(\alpha)\Phi_s^*(\alpha)}{\sqrt{k_0^2-\alpha^2}}\,d\alpha, \quad (14)$$

and $\Phi_n(\alpha)$ is the Fourier transform of the normalized mode function. Equation (14) demonstrates the essential modal coupling arising because of the fluid loading effects. The form of the coupling coefficient Z_{ns} has been discussed in great detail by Davies.[20]

Upon substituting for P_s into Eq. (8), the final equation for the unknown modal expansion coefficients is given by

$$(\omega_s^2-\omega^2)W_s=F_0\phi_s(x_0)+\sum_{n=1}^{N} W_n G_{ns}+\sum_{r=1}^{R} m_r'\sum_{n=1}^{N} H_{nrs}W_n, \quad (15)$$

where the following definitions have been made:

$$G_{ns}=(j\rho_0\omega^2/2\pi)Z_{ns} \quad (16)$$

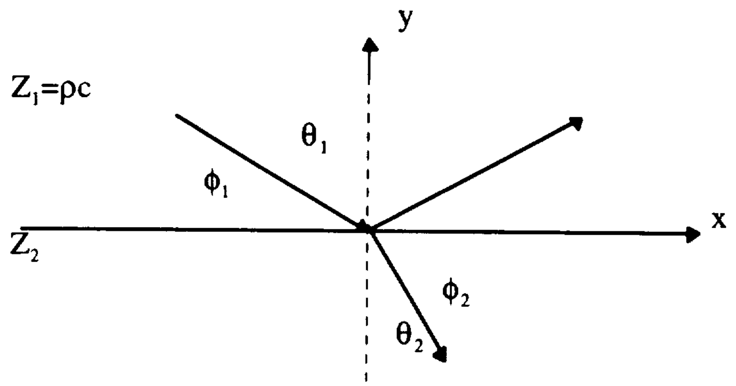
and

$$H_{nrs}=\omega^2\phi_n(x_r)\phi_s(x_r). \quad (17)$$

Problem I

Consider an harmonic plane wave incident at an angle θ_1 on an interface (y=0). Its complex amplitude is written as:

$$\hat{P}_i = \hat{P}_0 e^{ik_x x} e^{-ik_y y}$$



Two students are asked to derive an expression for the (pressure amplitude) reflection coefficient R. Student #1 writes $Z_2 = p/v_{in}$ at y=0, and derives an expression for the reflected and the total pressure in the upper medium, and derives

$$R = \frac{\sin\phi_1 - Z_1/Z_2}{\sin\phi_1 + Z_1/Z_2}$$

But student #2 understands the problem as a transmission problem into a second medium where the refracted wavenumber becomes k_2. He writes the continuity equations at the interface in terms of R and T (the amplitude reflection and transmission coefficients), and, using the trace velocity matching principle, derives an expression for the reflection coefficient in terms of the grazing angle ϕ_1

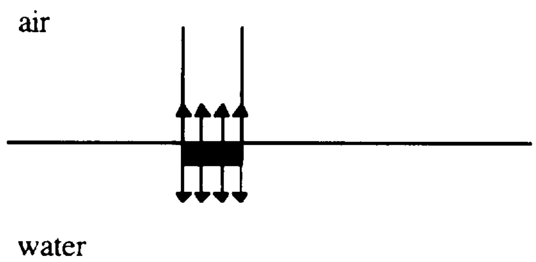
$$R = \frac{\sin\phi_1 - (Z_1/Z_2)\sqrt{1-(k_1/k_2)^2\cos^2\phi_1}}{\sin\phi_1 + (Z_1/Z_2)\sqrt{1-(k_1/k_2)^2\cos^2\phi_1}}$$

(a) Retrace the steps of student #1
(b) Retrace the steps of student #2
(c) Why are the resultrs different? Are they both correct?

Problem II.

When a high power laser pulse is focused on an air/water interface, it often produces a localized sudden impact force on the surface. The force is caused by the sudden evaporation and water ejection from the illuminated region into the upper medium (air). Conservation of momentum requires that a localized force of equal but opposite sign be imparted into the water. The force is distributed in a volume defined by the laser spot size on the water surface and the penetration depth of the laser light inside the water. Let this source term (per unit volume) be $f(t,\vec{R})=g(t)F(\vec{R})$, where $\vec{R}$ is a function of (x,y,z). [g(t) is related to the laser pulse shape (time) and $F(\vec{R})$ is related to the laser intensity distribution within the illuminated volume. In general, the absorption depth is very short compared to a typical acoustic wavelength.]

(a) From the basic conservation laws, derive a linearized acoustic wave equation for propagation in the lower fluid (water), including the force density term $f(t,\vec{R})$. (Neglect thermal expansion effects)



(b) Make an educated guess as to the nature of such a source, i.e. monopole / dipole / quadrupole type source. Justify your answer.