1. a) ***F*** represents a force and is given by

$$\boldsymbol{F} = -2(z - y)\mathbf{i} + 2x\mathbf{j} - 2x\mathbf{k}.$$

Find the single-valued function ϕ that is the potential of ***F***. Assume ***F*** is conservative.

b) ***F*** represents a force and is given by

$$\boldsymbol{F} = -y/(x^2 + y^2)\mathbf{i} + x/(x^2 + y^2)\mathbf{j}.$$

Is there a single-valued function that is the potential of ***F***? Show why or why not.

2. The Fourier series is a very powerful tool in representing periodic functions. If we already know the analytic form of the function, say $y = f(x)$, then we have a standard formula to obtain the coefficients in the Fourier series. In practice we truncate the series to a finite number of terms. If, however, we do not have the analytic form, we may need to do this numerically. Let us consider the following simplest problem of this kind. Suppose we have the data points $(x_i, y_i), i = 1,2,\cdots, N$. We need to fit this data to the following model:

$$y = A_0 + A_1 \cos(\omega_0 x) + B_1 \sin(\omega_0 x)\ .$$

Here, ω_0 is the fundamental frequency, which is known. We need to find the best choice of the constant coefficients A_0, A_1, B_1, in the least square sense. Assume the special case that $A_0 = 0$ (and hence we know *a priori* that there is no constant offset). Solve for the coefficients A_1 and B_1. Based on the results, discuss when this method does not work.

3. Solve the following initial value problem:

$$(t+1)\frac{dy}{dt} - ny = e^t (t+1)^{n+1}$$
$$y(0) = 1$$

4. A real nonsingular matrix is block partitioned as $\mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{B} & \mathbf{C} \end{bmatrix}$ where **A** and **C** are symmetric. The inverse of $\mathbf{M}^{-1}$ can also be block partitioned as $\mathbf{M}^{-1} = \begin{bmatrix} \mathbf{W} & \mathbf{X} \\ \mathbf{Y} & \mathbf{Z} \end{bmatrix}$.

   (a) Determine the matrix blocks of $\mathbf{M}^{-1}$ in terms of the matrix blocks of **M**.

   (b) To do part (a), it is necessary to make two important assumptions. State them and <u>prove</u> that they are true.

   (c) Find the determinants $|\mathbf{M}|$ and $|\mathbf{M}^{-1}|$ in terms of the matrix blocks.

   (d) Assume that $\mathbf{B} = \mathbf{0}$. Show that there exists a matrix **Y** such that $\mathbf{M} = \mathbf{Y}\mathbf{Y}^T$.