GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1999

Applied Math
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Instructions: Do four of the five following problems. Your answers should be as complete as possible. Extra sheets of paper are available upon request.

1. Given the symmetric matrix **A** with components

$$[\mathbf{A}] = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 3 & 2 \end{bmatrix}$$

(a) Determine the eigenvalues of **A.**

(b) Determine the unit eigenvectors of **A**.

(c) Is this set of eigenvectors linearly independent?

(d) Is **A** orthogonal?

(e) Determine $\mathbf{A}^{-1}$ (if possible) and directly solve the linear equation $\mathbf{A}\,\mathbf{x} = \mathbf{y}$ for the vector $\mathbf{x}$, where $\{y\} = \{0, 1, 2\}^T$.

2. Employ the Newton-Raphson method to determine a real root for:

$$f(x) = -2.0 + 6x - 4x^2 + 0.5x^3,$$

using an initial guess of $x = 0.5$.

(a) What is the error in your final result?

(b) Why does this method work?

3. Consider the temperature $u(x, t)$ in a thin bar of length L oriented along the x-axis as a function of time t. The one-dimensional heat equation is given by

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial t^2},$$

where c is a constant which depends on the material properties. Consider the following boundary conditions:

$$u(0, t) = 0 \text{ and } u(L, t) = 0,$$

and the following initial condition:

$$u(x, 0) = f(x) \text{ , where } f(x) \text{ is a known function.}$$

Solve for $u(x, t)$ by the method of separation of variables.

4. Consider the vector field

$$\mathbf{v}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k} \tag{1}$$

where x, y, and z are Cartesian coordinates and **i**, **j**, and **k** are respective unit vectors along x, y, and z axes.

(a) Find $\varphi(x, y, z)$ and $\mathbf{A}(x, y, z)$ such that

$$\mathbf{v} = \nabla\varphi + \nabla \times \mathbf{A} \tag{2}$$

(b) Are the scalar potential φ and vector potential **A** unique? Why or why not?

(c) For an arbitrary vector field **v**, do φ and **A** always exist for **v** to be expressed by equation (2)? Why or why not?

For (b) and (c), a brief answer and explanation is sufficient. A formal proof is not required.

5. Find the general solution of the following ordinary differential equation:

$$y''' - 6y'' + 11y' - 6y = 2xe^{-x},$$

where x is the independent variable.

Hints:

(1) Consider the method of variation of parameters for the particular solution.

(2) Note that $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = (\lambda - 1)(\lambda - 2)(\lambda - 3)$.