GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2002

Applied Mathematics
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

APPLIED MATHEMATICS QUALIFYING EXAMINATION

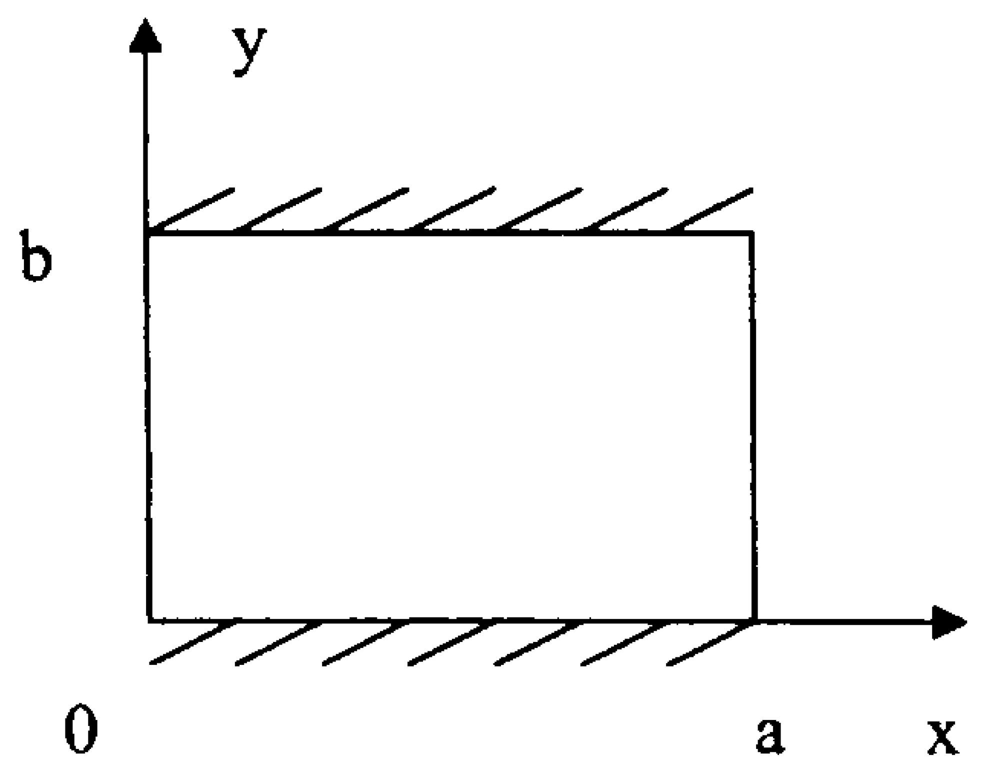
FALL 2002

Please answer ***all four*** of the following four questions!
Be sure to answer all parts of each question.
Show all of your work!

PROBLEM 1

SEPARATION OF VARIABLES:

The partial differential equation below [See eq. (1)] and the boundary conditions (2a and 2b) model the vibration amplitude of a rectangular membrane (dimensions a x b) that is clamped at $y=0,\ b$ and free at $x=0, a$, as shown in the picture below. The domain of interest is



$$0 \le x \le a$$
$$0 \le y \le b$$
$$t \ge 0$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \qquad (1)$$

where c is a real valued constant.

$$u \Big|_{y=0,b} = 0 \qquad \forall x, t \qquad (2a)$$

$$\left.\frac{\partial u}{\partial x}\right|_{x=0,\,a} = 0 \qquad \forall\, y, t \qquad (2b)$$

<u>a) Appetizer</u>: Is the PDE linear or nonlinear? Is it first order or second order? Justify your choices.

<u>b) Main course</u>: Use the method of separation of variables to find a general solution for $u(x,y,t)$. Define the eigen functions (in x and y) and the corresponding eigen values.

<u>c) Dessert:</u> Sketch the vibration mode shape for the lowest non-trivial mode.

PROBLEM 2

Use simple fixed-point iteration to locate a root of $f(x)$ using initial guess of $x_0 = 2$.

$$f(x) = -x^2 + 2x + 3 = 0$$

(a) If the function is formatted as

$$x = \frac{x^2}{2} - 1.5$$

Do you expect to achieve a convergent result? Explain the reason.

(b) Choose another form of $x = g(x)$ that results in a convergent fixed-point scheme. Perform the computation until ε_t is less 0.1%.

(c) Please suggest an alternative numerical method that converges faster than the fixed-point iteration and perform the computation until ε_t is less 0.1%.

(Note: please use $x_0 = 2$ as the initial guess in all calculations).

PROBLEM 3

Assume that A is a real $n \times n$ matrix.

(a) For $A = A^T$:

(i) If two eigenvalues are distinct, show their corresponding eigenvectors are orthogonal.

(ii) If two eigenvalues are the same, then completely explain the orthogonality and/or nonorthogonality of their corresponding eigenvectors.

(b) Give an example of a matrix A that does not have a complete set of eigenvectors and show that it is true.

(c) Show that every matrix A satisifies its own characteristic equation.

PROBLEM 4

Consider a system of n ordinary differential equations (ODEs)

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A} \cdot \mathbf{x}(t) \quad ,$$

where **A** is a $n \times n$ constant matrix and $\mathbf{x}(t)$ is a column vector consisting of the n unknowns, $x_1(t)$, $x_2(t)$, ... $x_n(t)$. Assume that **A** is NOT a diagonal matrix. Please STATE and PROVE a sufficient condition under which the above system of n ODEs can be transformed into n decoupled first order ODEs (Hint, consider the eigenvalues and eigenvectors of **A**).