GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - FALL Semester 2001

Applied Math
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Applied Mathematics Ph.D. Qualifying Exam – Fall, 2001

Work FOUR of the following five problems. Only four problems will be graded—do NOT do all five!

Problem 1

a) A rhombus is a parallelogram with equal sides. If the length of a side is a and one of the rhombus angles is 2θ, use a vector formulation to determine the angles at which the rhombus diagonals intersect.

b) If a vector is given an arbitrary three-dimensional rotation, analytically show that it s length is unchanged.

c) A constant force applied to a rigid body creates a moment about each point and thus a vector field of moments. Determine the curl of this field.

d) Find the angle between two surfaces $x^2 + y^2 = 9 - z^2$ and $x^2 + y^2 = z + 3$ at the point (2, -1, 2).

Problem 2

A general solution of a nonhomogeneous linear ordinary differential equation is a sum of the form

$$y = y_h + y_p$$

where y_h is a general solution of the corresponding homogeneous equation, y_p is any particular solution of the nonhomogeneous equation. In general, most of the problems in finding y lies on finding y_p. In any standard textbooks, there are many discussions of this subject.

Consider a spring-mass system under an input driving force $r(t)$:

$$my''(t) + ky(t) = r(t)$$

where $m > 0$ is the mass, $k > 0$ is the spring constant, t is the time (the independent variable), $y(t)$ is the position of the mass, and $y''(t) \equiv d^2y(t)/dt^2$. In case you are not familiar with a system of this kind, just ignore the physical meaning and treat the rest simply as a mathematical problem.

Let $\omega_0 = \sqrt{k/m}$ denote the natural frequency of the system. Consider the input driving force to be

$$r(t) = F_0 \cos \omega_0 t$$

where $F_0 > 0$ is a constant. In other words, the frequency of the input driving force is the natural frequency. In physical term, we call this the case of *resonance*.

Obtain y_h and y_p (and hence y) for this case. Assume the initial condition $y(0) = k_1$ and $y'(0) = k_2$.

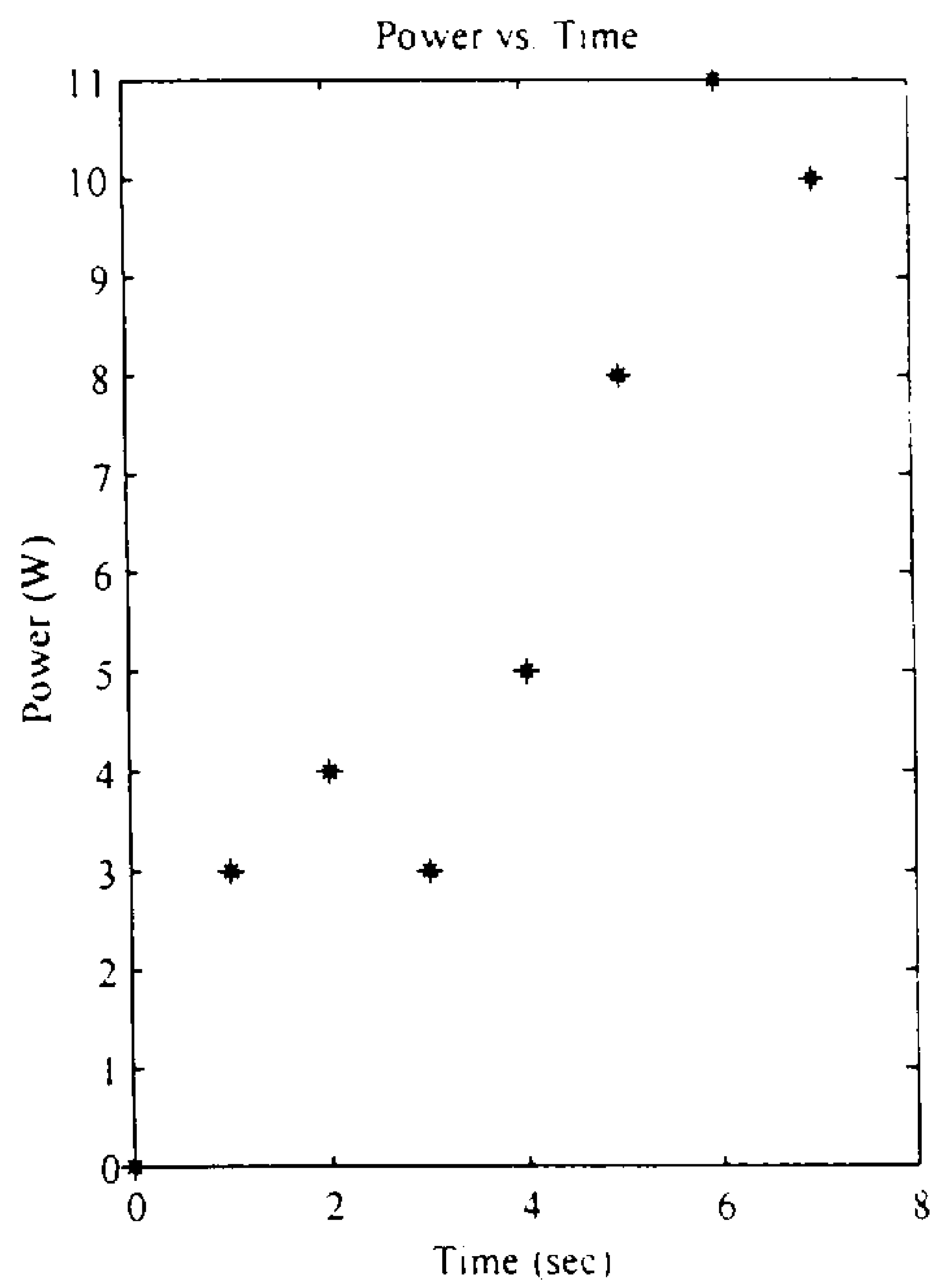
Very briefly discuss the behavior of $y(t)$ as $t \to \infty$.

Problem 3

Consider the experimental power vs. time data set below. You are to integrate these data numerically. using the trapezoidal rule and a combination of **Simpson's 1/3 and 3/8 Rules**. See the graph.

Power (W)	0	3	4	3	5	8	11	10
Time (sec)	0	1	2	3	4	5	6	7

a) Integrate the data using the **trapezoidal rule**.



b) Simpson's 1/3 and 3/8 Rules: Briefly explain your approach to solving this problem using a combination of Simpson's 1/3 and 3/8 rules. That is, which segments will be integrated using Simpson's 1/3 rule? Which segments will be integrated using Simpson's 3/8 rule?

c) Now, perform the integration using Simpson's 1/3 and 3/8 Rules.

d) Compare your results and comment on them using your knowledge of numerical integration methods and their errors.

Problem 4

Solve the following boundary value-initial value problem using the separation of variables approach.

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{k}\frac{\partial w}{\partial t} \qquad 0 < x < a, \qquad 0 < t$$

$$w(0, t) = 0, \qquad 0 < t$$

$$\frac{\partial w}{\partial x}(a, t) = 0, \qquad 0 < t$$

$$w(x, 0) = f(x), \qquad 0 < x < a$$

Problem 5

Consider a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$, described by (3×3) matrix $[A]$. Furthermore, let $[A]$ be length-preserving, i.e. for all vectors $\mathbf{x} \in \mathbb{R}^3$, the length of $\mathbf{x}$ is equal to the length of $([A]\mathbf{x})$.

(a) (50 % of points)
Determine all possible values of the eigenvalues, λ_i, for the considered matrix, $[A]$.

(b) (25 % of points)
Show that $[A]$ is angle-preserving in the following sense:
For all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$ *it holds that the magnitude of the angle between* $\mathbf{x}$ *and* $\mathbf{y}$
$=$ *magnitude of the angle between* $[A]\mathbf{x}$ *and* $[A]\mathbf{y}$.

(c) (25 % of points)
Prove or disprove (by counter-example) the following statement:
For the considered matrix $[A]$, *it must hold* $\det([A]) = 1$.