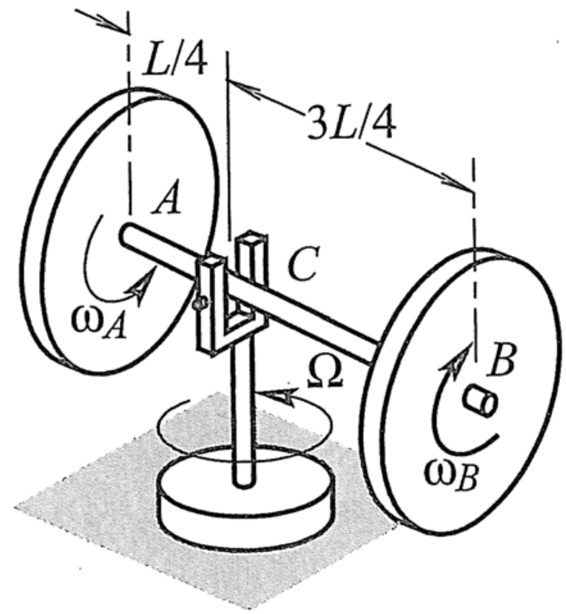
Dynamics and Vibrations Qualifying Examination

Spring 2010

Instructions: Work 3 out of 4 problems. If you submit work for all 4 problems, only the first 3 will be graded.

There is an equation sheet at the back of the exam.

1) Identical thin disks A and B, each of mass m and radius R, spin at the constant rates ω_A and ω_B, respectively, about massless shaft ABC, which is horizontal. The disks spin freely about this shaft without friction. A horizontal pin at C passes through the shaft, and the shaft is free (without friction) to rotate about this pin. Note that the disks are **not centered** about the pin C, but instead placed distances $L/4$ and $3L/4$ from the pin. No external forces or moments are applied to the system, except gravity g.



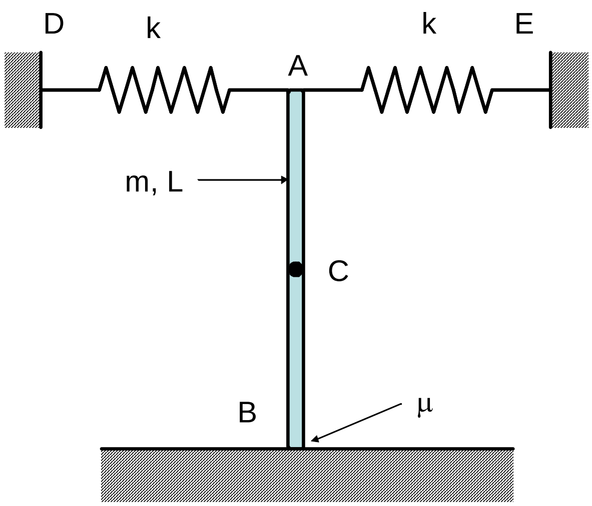
a) If the entire system precesses about the vertical axis at a constant rate Ω while the shaft ABC remains horizontal, determine the reactions (forces and moments) at C and the relationship between ω_A , ω_B, g, L, R, and Ω.

b) If an external torque is applied to the vertical shaft of magnitude T and oriented in the vertical direction, show that it is impossible for the shaft ABC to remain horizontal. You may elect to assume the shaft ABC remains horizontal and then show that this condition leads to contradictory conclusions from the equations governing changes in angular momentum.

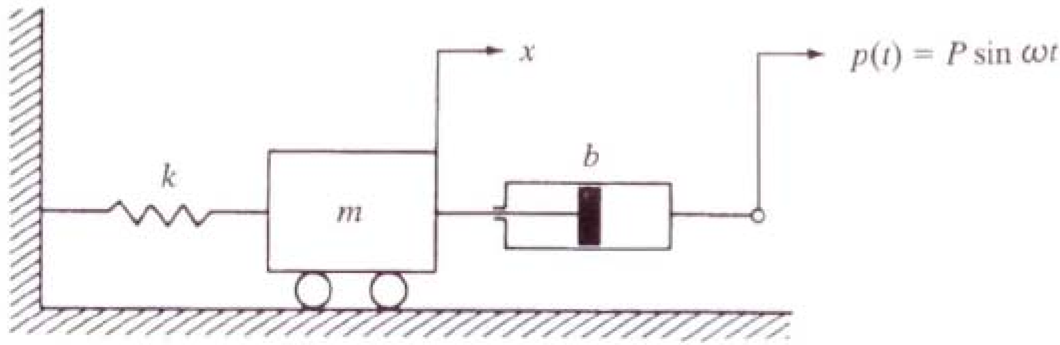
2) A uniform, slender rod of length L and weight $W = mg$ is at rest in the vertical position as shown. Each spring has been stretched so that the tension in each spring is P. The coefficient of friction between the rod and the ground is μ. The moment of inertia of the rod about its mass center, C, is given by $I_C = (mL^2)/12$.

(a) If spring AD suddenly breaks, give the relationship between P, μ, W, and L such that the end B slides to the left.

(b) For $\mu = 0.3$ and $P = 1.6W$, find the acceleration of point B immediately after spring AD breaks.



3) The mechanical system shown below comprises a spring (stiffness k), mass (m) and dashpot (viscous friction coefficient b) attached in series. Assume that the input is displacement p and the output is displacement x. Assume that the system remains linear throughout the operating/response period. The displacement x is measured from the equilibrium position.



3.1) Suppose that the displacement p is harmonic, i.e. $p(t)=P\ sin(\omega t)$.

a) What is the exact expression of the time-domain output $x(t)$ at steady state?

b) **Sketch** the temporal behavior of the response $x(t)$ with respect to the input signal $p(t)$ for both very low ($\omega \rightarrow 0$) and very high ($\omega \rightarrow \infty$) input frequencies. Specify the phase and magnitude of $x(t)$ for these two asymptotic cases.

3.2) Suppose now that the system is initially at rest, and that at t=0, a unit-step displacement input $p(t)$ (i.e. "Heaviside" function) is applied.

a) What is the exact expression of the time-domain output $x(t)$

b) If m=1kg, b=10 N-s/m and k=50 N/m, give the values of $x(t=0+)$, dx/dt (t=0+) (i.e. the initial displacement and initial velocities immediately after the unit-step is applied), and $x(t=\infty)$ (i.e. the displacement for large time values)

HINTS:

1) the Heaviside function $u(t)$ and the Dirac Delta (impulse) function $\delta(t)$ are related by the following relationships:

$$\int_{-\infty}^{t} \delta(u-a)du = u(t-a) = \begin{Bmatrix} 0 & if\, t < a \\ 1 & if\, t > a \end{Bmatrix}$$, and

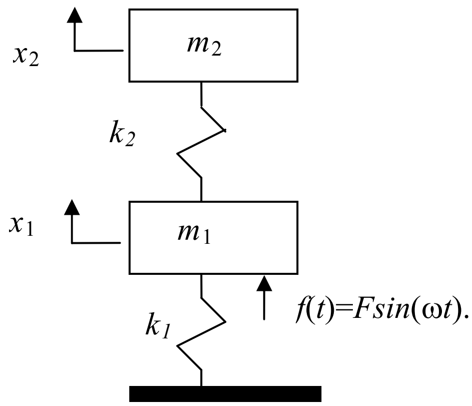
$$\frac{d}{dt}u(t-a) = \delta(t-a) = \begin{Bmatrix} +\infty & if\, t = a \\ 0 & if\, t \neq a \end{Bmatrix}$$

2) The following table of Laplace Transforms may be useful. s denotes the complex variable of the Laplace transform (or $s=j\omega$ for the Fourier Transform).

TABLE 1: Laplace Transform Pairs.

$f(t)$	$F(s)$
$\delta(t)$, impulse at $t = 0$	1
$u(t)$, Step at $t = 0$	$1/s$
$t[u(t)]$, Ramp at $t = 0$	$1/s^2$
$e^{-\alpha t}$	$1/(s+\alpha)$
$te^{-\alpha t}$	$1/(s+\alpha)^2$
$e^{-\alpha t}\sin\beta t$	$\beta/\left[(s+\alpha)^2+\beta^2\right]$
$e^{-\alpha t}\cos\beta t$	$(s+\alpha)/\left[(s+\alpha)^2+\beta^2\right]$
$\dfrac{d^n f(t)}{dt^n}$	$s^n F(s) - \displaystyle\sum_{m=0}^{n-1} s^{n-m-1}\dfrac{d^m f(0)}{dt^m}$
$\int_0^t f(\tau)\,d\tau$	$(1/s)\,F(s)$
$\int_0^t x(t-\tau)\,h(t)\,d\tau$	$H(s)\,X(s)$
$f(t-\tau)$	$e^{-\tau s}F(s)$
$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$
$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$

4) Consider the system depicted below. The disturbance f applied to x_1 is harmonic, such that $f(t)=F\sin(\omega t)$.

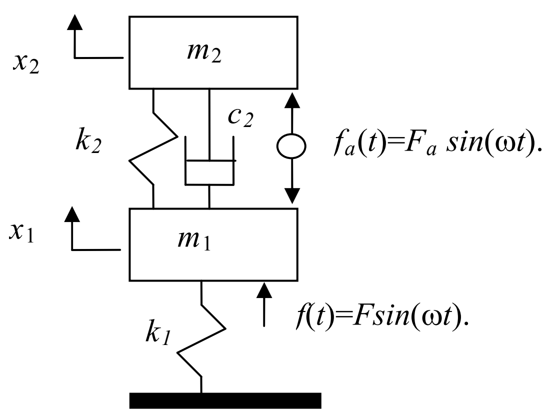


a) Express the equations of motion for this system in matrix form, $Z\underline{x}=\underline{f}$ where $\underline{x}=\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$ and is a vector of forces on body 1 and 2, respectively. Suppress the harmonic time dependency.

b) Solve the system of equations for the displacement x_1. (If you use matrix methods, you may find it useful that $Z^{-1}=\dfrac{1}{Det(Z)}\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$, where $Det(Z)=z_{11}z_{22}-z_{21}z_{12}$.

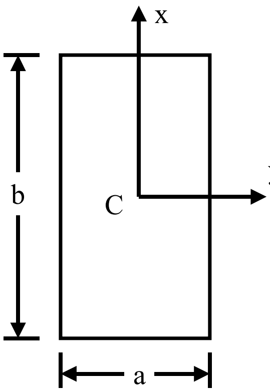
c) Under what conditions will the displacement x_1 be minimum? (you don't need calculus)

d) Consider the modified system below, where a damper and a force actuator have been placed between the masses. The force actuator exerts equal but opposite forces onto both masses. What must be f_a in order for x_1 be minimized for all frequencies? (you don't need calculus)



Equation Sheet For Dynamics and Vibrations PhD Qualifying Exams

Centroidal inertia properties for thin plate



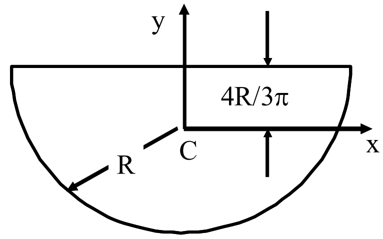
$I_{xx} = ma^2/12$

$I_{yy} = mb^2/12$

$I_{zz} = m(a^2 + b^2)/12$

$I_{xy} = I_{xz} = I_{yz} = 0$

Centroidal inertia properties for semicylinder



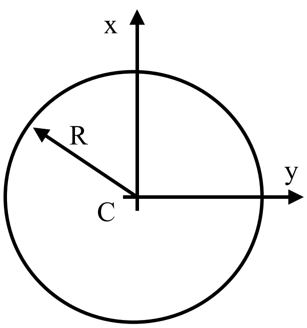
$$I_{xx} = \left(\frac{1}{4} - \frac{16}{9\pi^2}\right) mR^2$$

$$I_{yy} = \frac{mR^2}{4}$$

$$I_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right) mR^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Centroidal inertia properties for thin disk



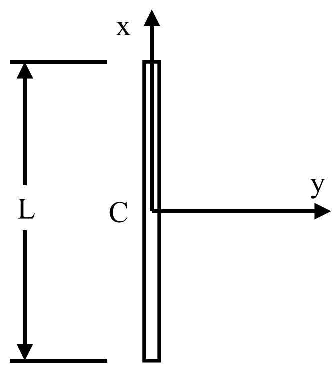
$I_{xx} = mR^2/4$

$I_{yy} = mR^2/4$

$I_{zz} = mR^2/2$

$I_{xy} = I_{xz} = I_{yz} = 0$

Centroidal inertia properties for slender rod



$I_{xx} = 0$

$I_{yy} = mL^2/12$

$I_{zz} = mL^2/12$

$I_{xy} = I_{xz} = I_{yz} = 0$

Beam Flexural Vibration:

Elastic modulus E, cross-sectional area $A(x)$, cross-sectional area moment $I(x)$, density ρ, length L, transverse displacement $w(x,t)$

$$T = \frac{1}{2}\int_0^L \rho\, A(x) \left(\frac{\partial w}{\partial t}\right)^2 dx; \quad V = \frac{1}{2}\int_0^L E\, I(x) \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx$$