Dynamics and Vibrations Qualifying Examination

Spring 2008

Instructions: Work 3 out of 4 problems. If you submit work for all 4 problems, only the first 3 will be graded.

Problem 1

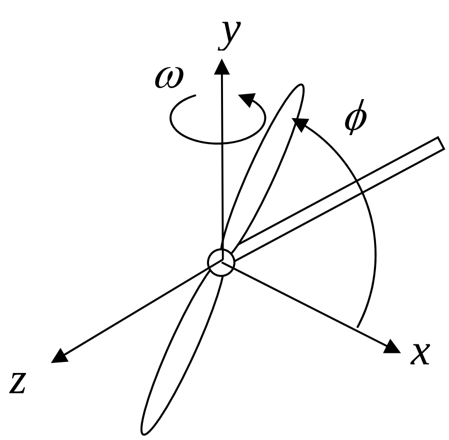
An airplane propeller is modeled as a *thin* homogeneous bar with principal moments of inertia (0, 0, I). An x, y, z coordinate system is attached to the airplane (not shown), the angle between propeller and the x axis is ϕ, and the propeller turns at a constant speed. A shaft connects the propeller to the airplane which is turning about the vertical y axis with constant angular velocity ω.

(a) Determine the moment applied on the propeller.

(b) Determine the bending moment applied on the shaft by the propeller and sketch it on the figure indicating the magnitude and direction.

(c) Determine the torsional moment applied on the shaft by the propeller and sketch it on the figure indicating the magnitude and direction.

Hint: You can work the problem using either propeller-fixed axes or airplane-fixed axes.

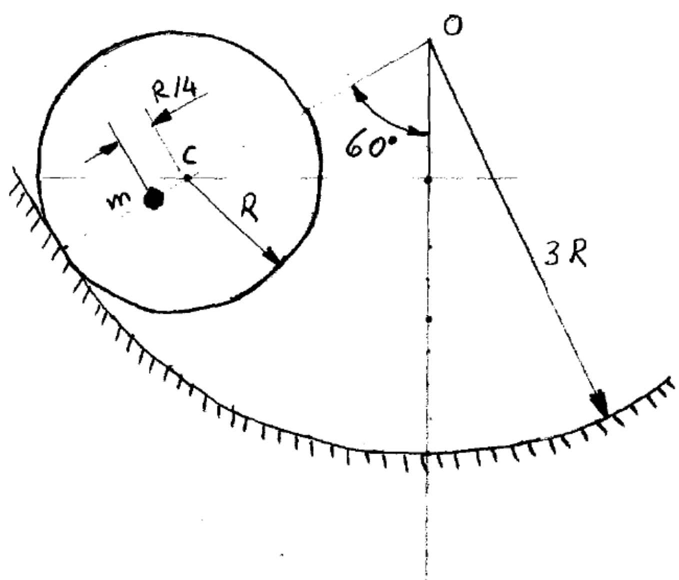


Problem 2

A particle of mass *m* is positioned at a distance *R/4* away from the center of a massless wheel, which is designated by point *C* in the figure. The wheel, of radius *R*, is released from rest from the position shown in the figure, allowing it to roll under the action of gravity inside a circular track of radius *3R*. At the instant of release (shown in the figure) points *O*, *C*, and *m* lie on the same line, which is inclined 60^o from the vertical axis. Gravity is acting downwards along the vertical axis.

A. Determine the maximum velocity that the mass *m* attains some time after release. Specify the location where this condition occurs and draw schematically its direction (that is, give the maximum velocity as a vector -- magnitude and direction).

B. Derive the differential equation of motion for wheel oscillation about the vertical axis [you do not need to solve this equation!].

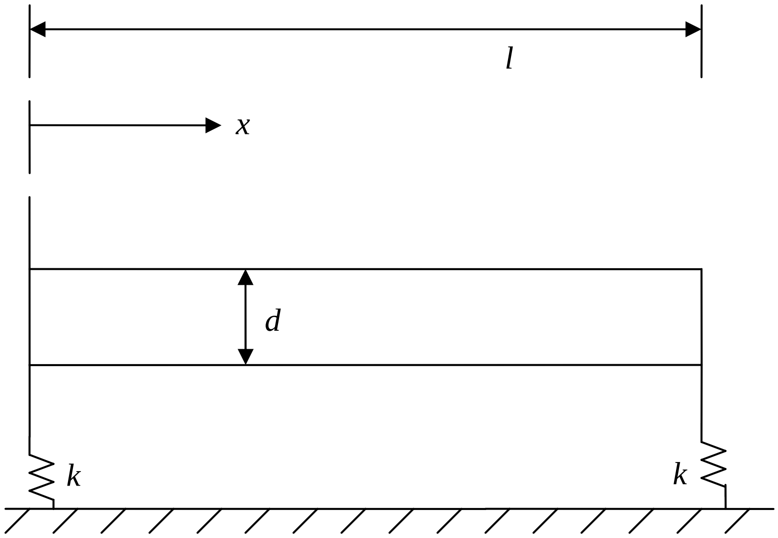


Problem 3

A machine weighting 664 lbs is mounted on a vibrating support that has a sinusoidal motion of frequency ω = 30 Hz and amplitude X = 0.05 in. The maximum acceleration of the machine is to be limited to 0.25g where g = 32.2 ft/s^2.

a) Determine the stiffness of the support.
b) Sketch a frequency response of the machine acceleration and indicate the operating point. (Use dimensionless ratios if possible.)

Problem 4



Given: The **circular** beam has a diameter d, a mass density ρ, a length l, and is composed of a material with Young's Modulus E.

Treat the beam using Euler-Bernoulli assumptions such that the moment at any location x is given by $M = EIw''$ where w'' is the curvature for small displacements in the vertical direction. Neglect rotary inertia - inertia associated with rotation of the x-section.

Find:

a) The equations of motion and boundary conditions governing small displacements $w(x,t)$ away from the static equilibrium position.

b) If the spring stiffness k is very small, what would you expect the first two vibration modes of the system to be?