GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2002

Dynamics and Vibrations
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2002

Instructions:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

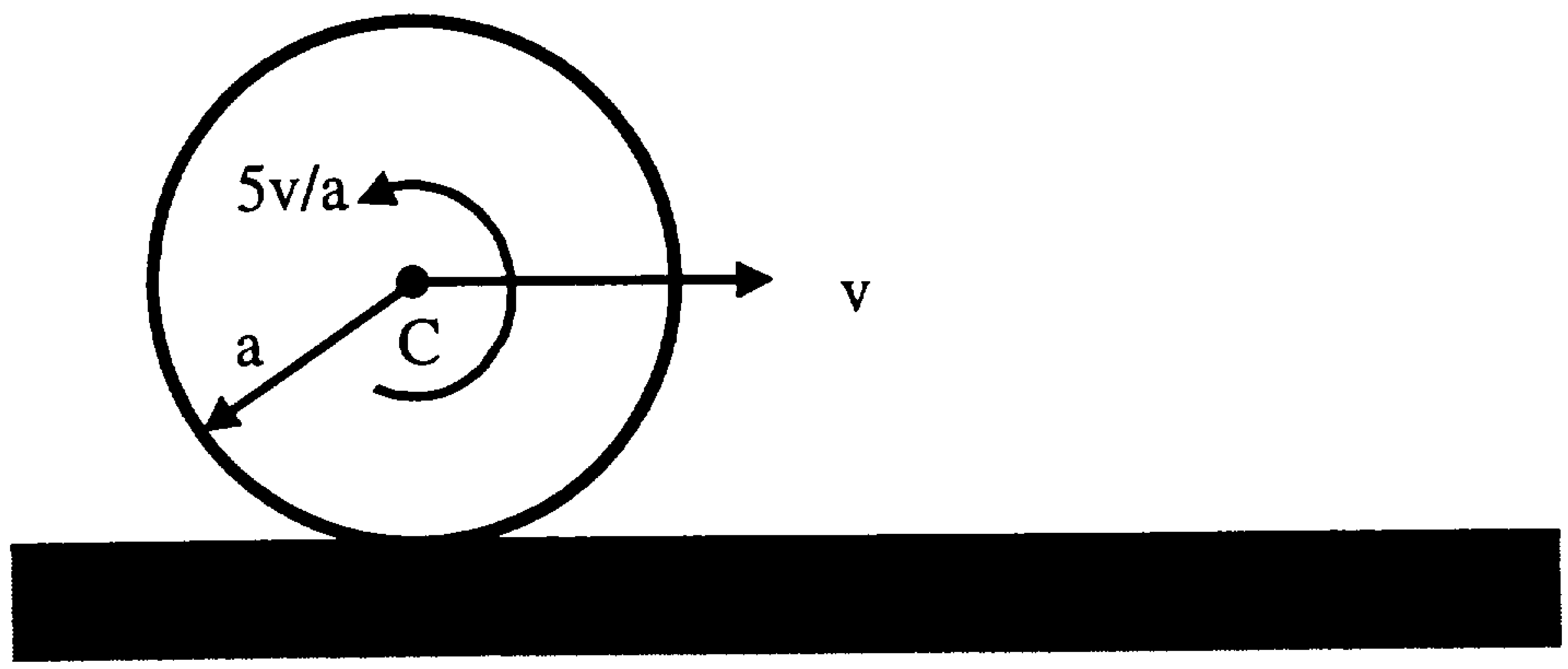
Problem 1.

A uniform circular disk of radius a, mass m, and mass-moment-of-inertia $I_C = ma^2/2$ is projected from left to right along a rough horizontal plane. The initial speed of the disk's center of mass is v and the initial angular velocity is $\omega = 5v/a$ counterclockwise. The kinetic coefficient of friction between the disk and the rough surface is μ.

(a) Find the initial acceleration (translational and angular) just after the disk is projected.

(b) Find the position of the disk's center of mass (relative to the starting location) at the time when the disk stops slipping and begins rolling.

(c) Determine the subsequent motion of the disk after rolling begins.



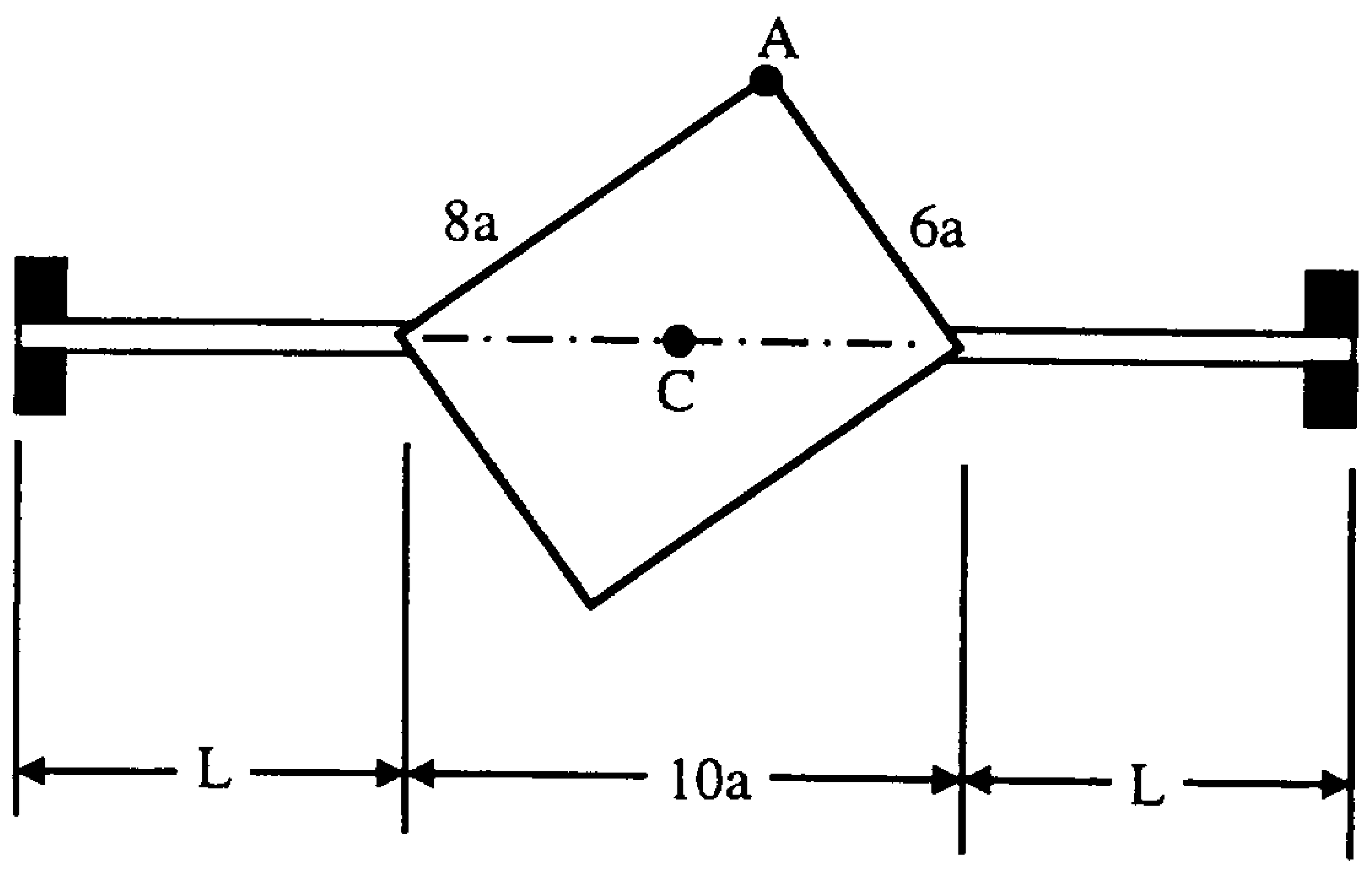
Problem 2.

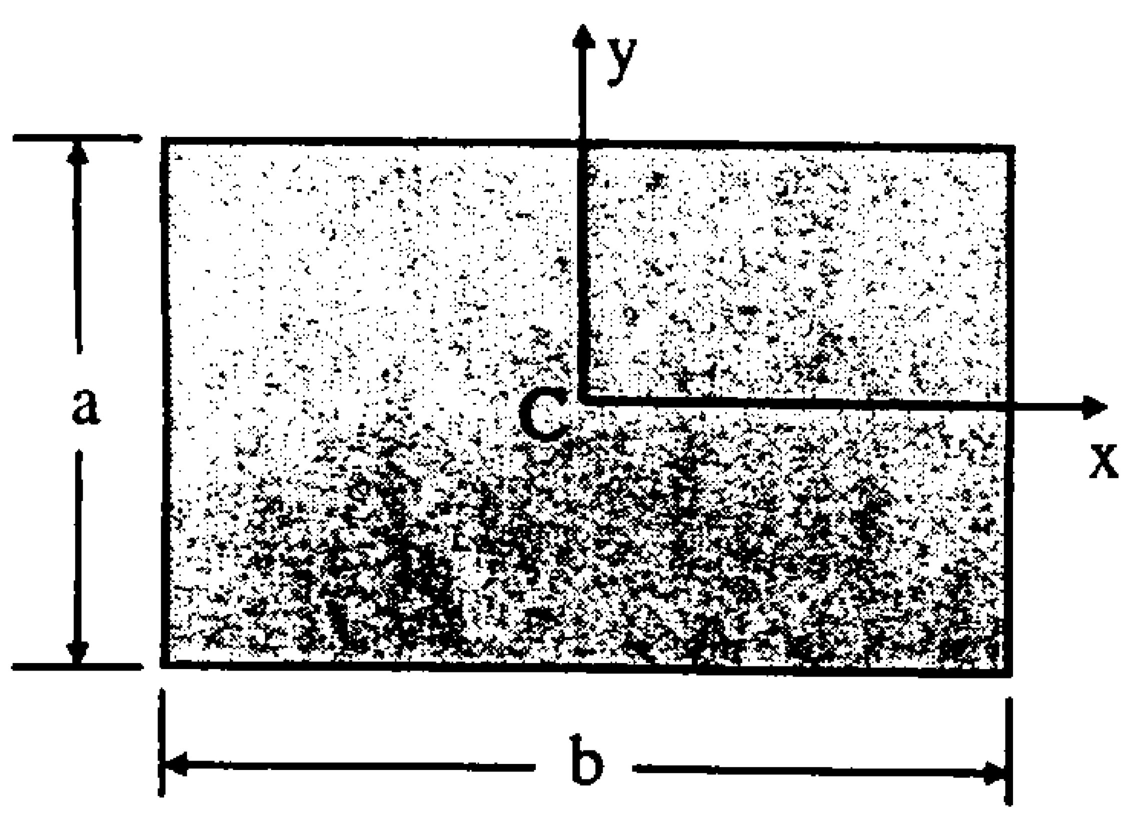
A thin rectangular plate of mass m and sides of length $6a$ and $8a$ is welded to a massless, rigid, horizontal shaft. The shaft rests in frictionless bearings. Initially, the plate is at rest in the vertical plane. At time $t = 0$, a projectile having mass $m_P = m/10$ strikes the plate at corner A. Before impact, the projectile is traveling horizontally in a direction perpendicular to the plate (into the page in the figure below) with velocity v; after impact, the mass sticks to corner A of the plate. The inertial properties of a thin rectangular plate are shown below. **Be sure to state your assumptions and to define any coordinate systems that you use in your solution.**

(a) Find the angular velocity of the plate just after the projectile impacts the plate. Assume that the plate's inertial properties are essentially unchanged from their values before the projectile becomes imbedded in the plate.

(b) After the impact is completed, what are the dynamic reactions at the left and right shaft bearings? If you are not able to complete part (a), denote the angular speed of the plate just after impact as ω_2.

(c) Give an expression for the energy lost in the collision process.





$$I_{xx} = \frac{1}{12} m a^2$$

$$I_{yy} = \frac{1}{12} m b^2$$

$$I_{zz} = \frac{1}{12} m \left(a^2 + b^2\right)$$

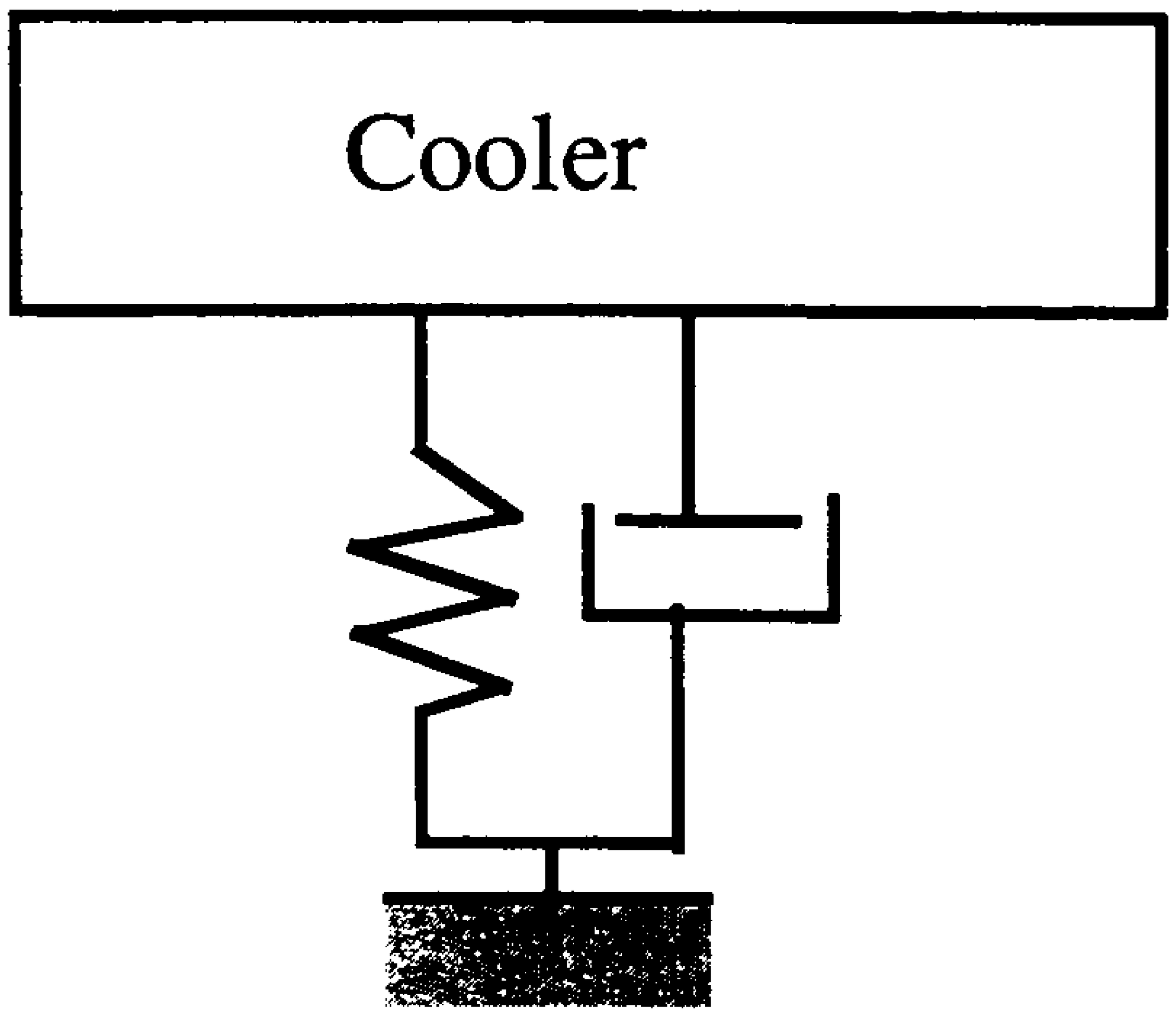
Problem 3.

An industrial vibratory cooler may be modeled by the system depicted in the figure below. The cooler has a mass of 2000 kg, and is supported on elastomeric springs of total spring constant 200,000 N/m and damping constant 1000 N/m/s. The vibratory action is induced by an internal mechanism that operates at 800 RPM. In operation, the cooler is observed to vibrate with an amplitude of 0.02 m.

(a) Determine the magnitude of the oscillating force produced by the internal mechanism.

(b) What is the magnitude of the force transmitted to the cooler's supports?

(c) How much mass would need to be added to the cooler in order to reduce its vibration amplitude to 0.01 m?



Problem 4

The system shown below is a triple pendulum, in which bars having mass m are pivoted from the ceiling and interconnected by springs k. The moment of inertia of a bar about its pivot is $\frac{1}{3}mL^2$.

(a) Derive the equations of motion for small oscillations of this system.

(b) *Without solving the eigenvalue problem for free vibration*, determine two mode vectors. These need not be normalized.

(c) Prove that the form of the third mode vector is

$$\{\phi_3\} = \left\{ \begin{matrix} a \\ b \\ a \end{matrix} \right\}$$

Then determine the ratio b/a.

(d) Determine the natural frequency associated with the third mode determined in Part (c).

(e) Explain how the eigenvalues and eigenvectors would be affected qualitatively if the value of k were increased with all other parameters held constant.

(f) Suppose that the mass of the center bar was $2m$, and all other parameters were unchanged. Which, if any, of the mode vectors for the equal mass case would be altered?