GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2001

Dynamics & Vibrations
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2001

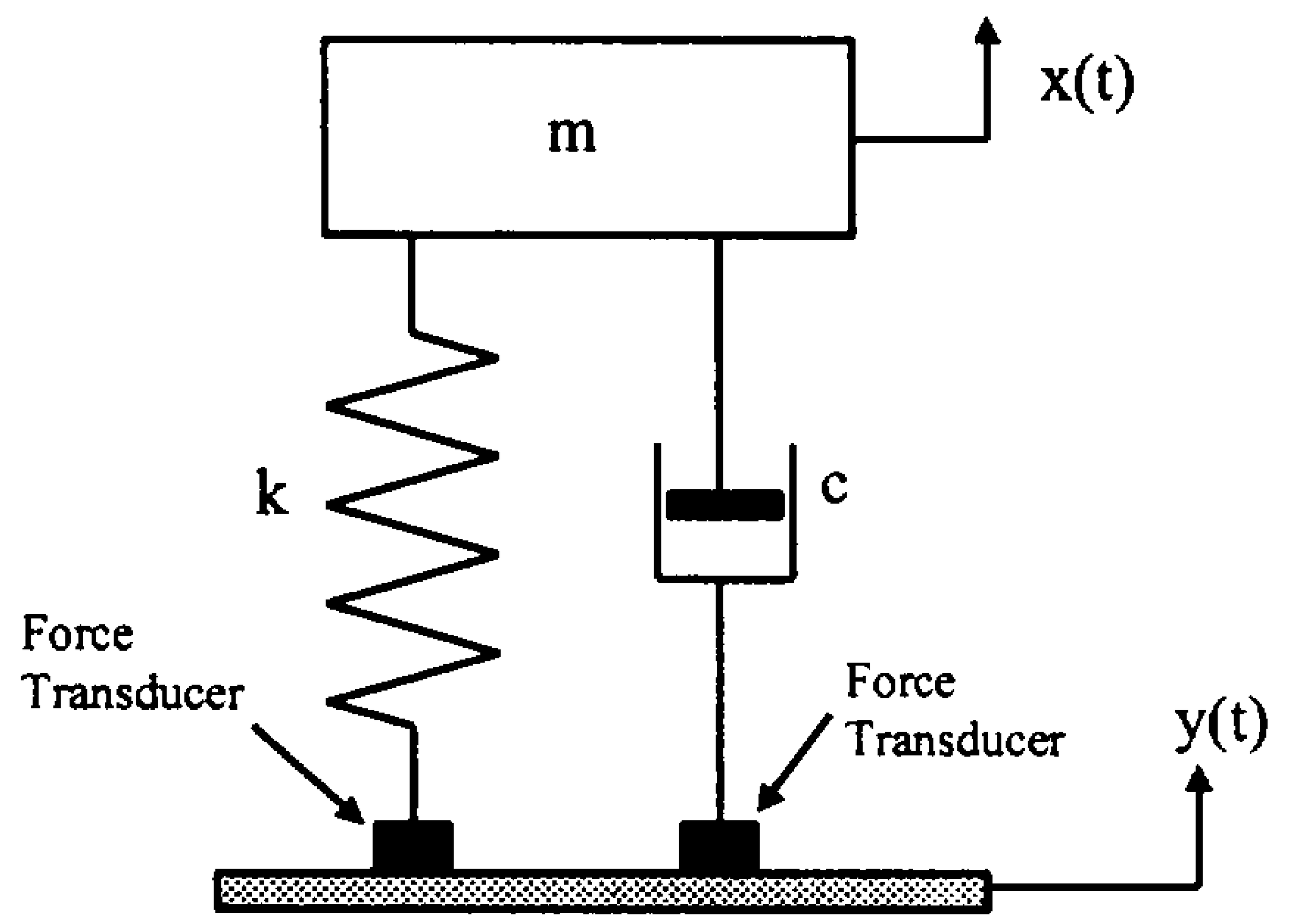
Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

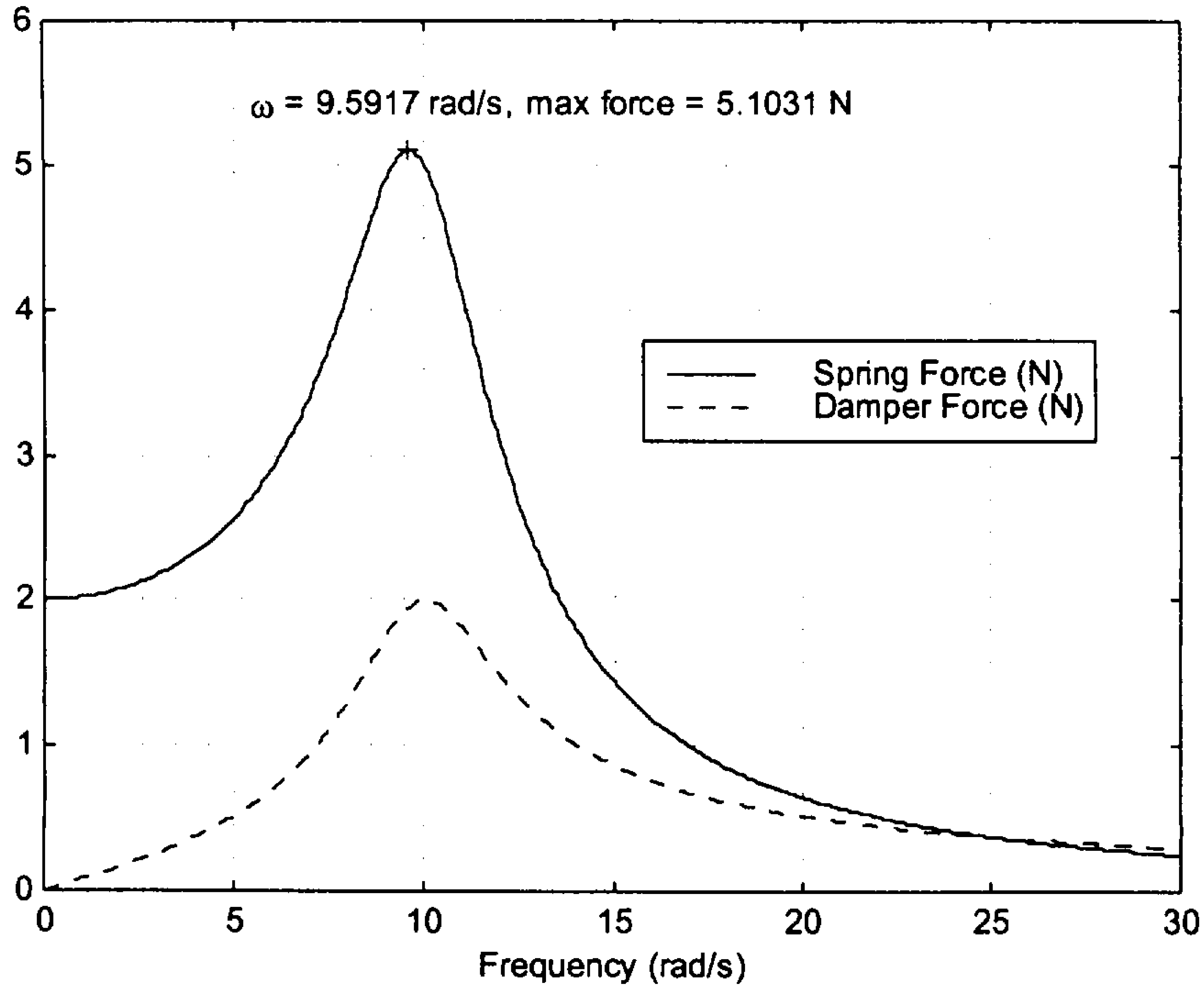
Problem 1.

Consider the base-excited spring-mass-damper system shown below, where force sensors (transducers) have been placed in series with the spring and viscous damper element. The plot below shows the frequency response magnitude plots for the spring-force sensor and the damper-force sensor as the system is subjected to a uniform 1-g *acceleration* input amplitude over all frequencies.

(a) Use the plots to determine the mass, m, the stiffness, k, and the viscous damping constant, c, for this system. Clearly state what assumptions if any you are using.

(b) As you know, by measuring the relative displacement between the mass and the moving base, the spring-mass-damper system can be used as the basis for an accelerometer or a seismometer. It has been suggested that, in a similar manner, the *damper-force signal* can be used to devise a *velocity sensor*. Discuss whether such a possibility exists, and if so, over what frequency range would the resulting "velocity-meter" be effective?



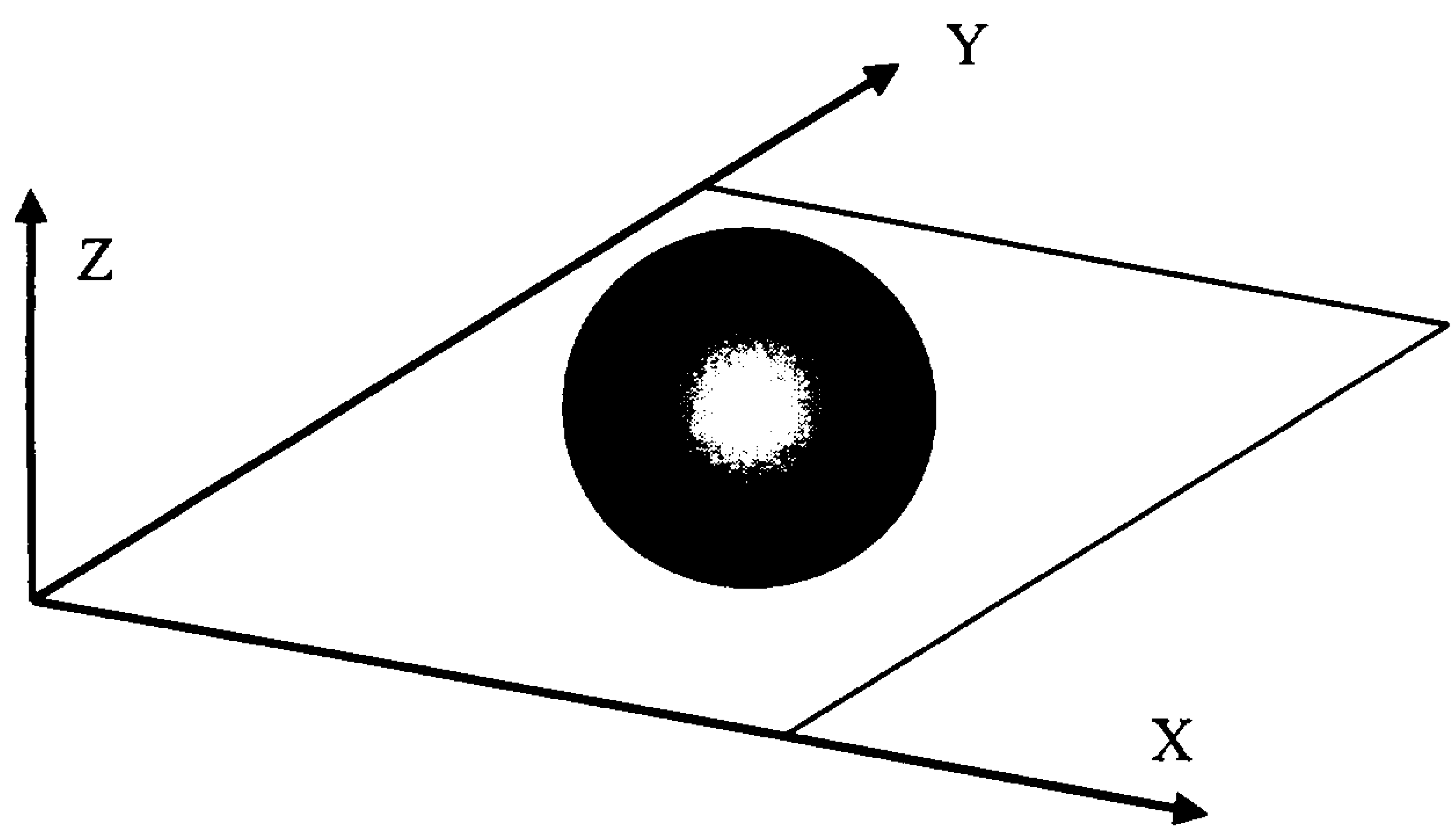


Problem 2.

A homogeneous sphere of mass m and radius R rolls without slipping on a rough horizontal plane as shown. In addition to the sphere's weight, there are applied forces P_X, P_Y, and P_Z acting at the sphere's center in the fixed X, Y, and Z-coordinate directions, respectively. Let the angular velocity of the sphere be given as $\vec{\omega} = \omega_X \vec{I} + \omega_Y \vec{J} + \omega_Z \vec{K}$, and let the linear velocity of the sphere's mass center be given by $\vec{v}_C = \dot{X}\vec{I} + \dot{Y}\vec{J} + \dot{Z}\vec{K}$ where $\vec{I}$, $\vec{J}$, and $\vec{K}$ are unit vectors in the space-fixed X, Y, and Z directions. The mass moment of inertia of the sphere is $I = (2/5)mR^2$.

(a) Find the scalar relationships that exist between the angular velocity components and the components of the linear velocity of the sphere's center of mass.

(b) Show that the equations of motion governing the horizontal motion of the mass center of the sphere are equivalent to those of a particle of somewhat greater mass in contact with a *smooth* horizontal surface.

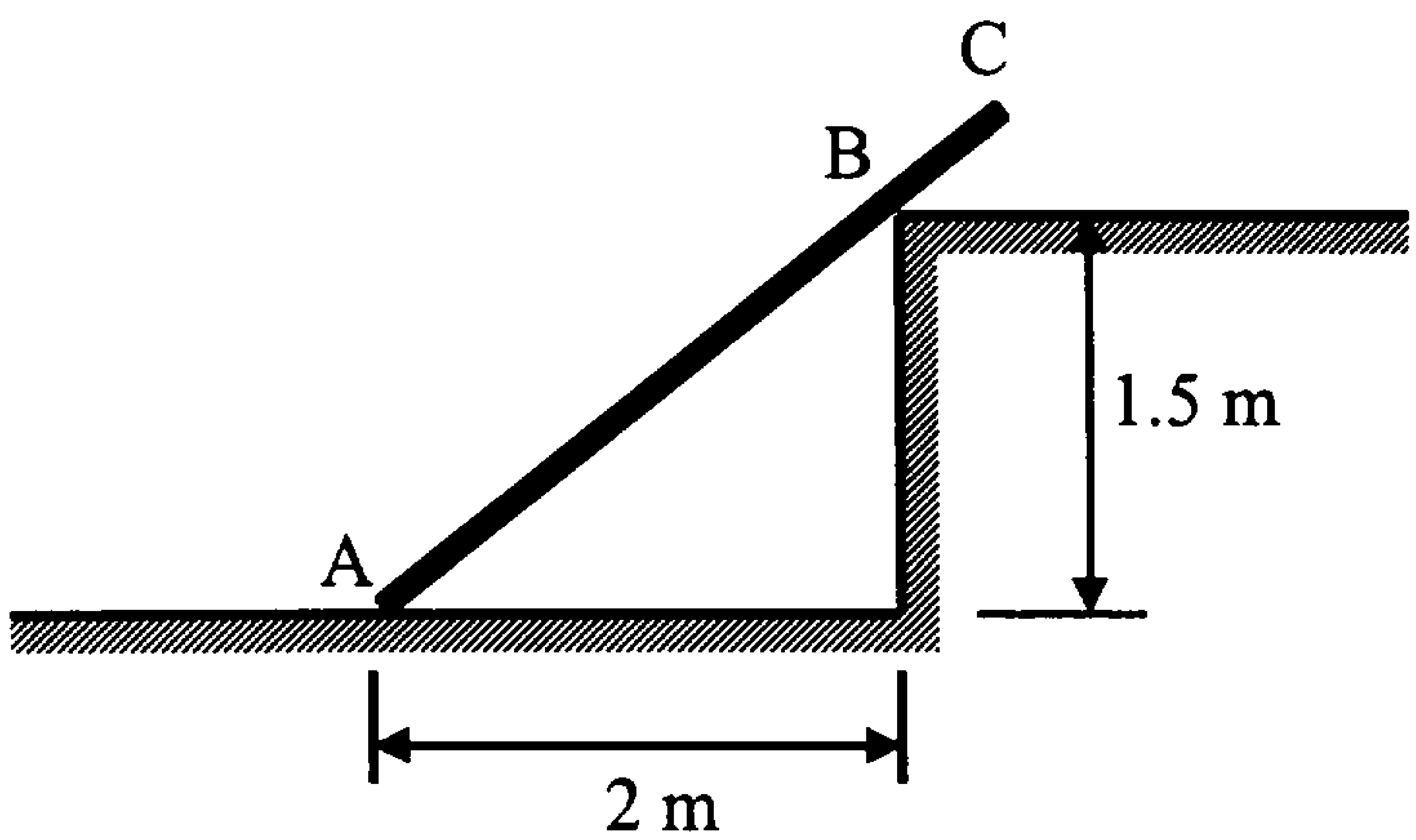


Problem 3.

A thin rod of length 3 meters rests on a rigid step as shown. Assume that there is no friction between the rod and any of the stationary surfaces and that contact between the rod and the stationary surfaces is not lost until after end C slides beyond the upper corner. Recall that the centroidal mass moment of inertia of a slender rod of length L and mass m is $I=(1/12)mL^2$. In both parts (a) and (b), concentrate on setting up the correct equations and solve them only if time permits.

(a) What is the initial acceleration of point A after the rod is released from rest?

(b) Assuming that the rod starts from rest, find the velocity of point A just before point C falls off the corner.



Problem 4.

A long slender rod of length $2a$ is suspended as shown and excited by means of "base excitation" y(t). The movement of point O is assumed to occur only in the horizontal direction.

(a) Assuming small values of the absolute centroidal displacement x(t) and the (absolute) rotation θ(t), *derive* the linearized equations of motion of the system. Recall that the mass moment of inertia of a slender rod about its center of mass is $I = (1/12)mL^2$.

Assume for parts (b), (c), and (d) of this problem, that the equations of motion are as given below.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} y(t)$$

(b) Find the natural frequencies and natural modes (eigenvectors) for the system.

(c) For the case where y(t) is harmonic, y(t)=Ysin(ωt), find the corresponding steady-state response amplitudes, X and Θ. Make a sketch of X and Θ vs ω being careful to note any points of "large" or zero response. What are the limits of X and Θ as ω goes to 0 or ∞?

(d) Is there any frequency for which point O is stationary in absolute space? If so, find that frequency.