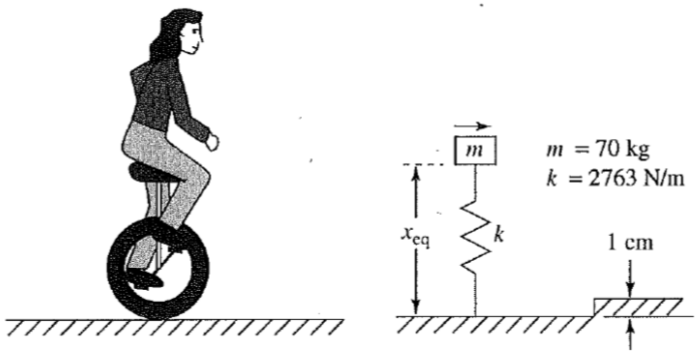
Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2011

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

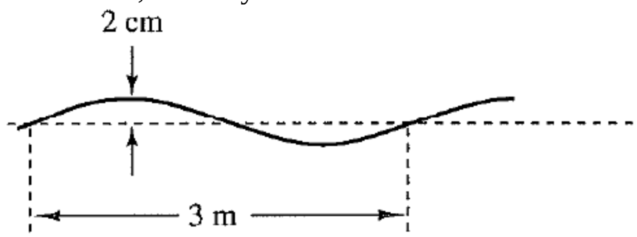
Problem 1

Consider the motions of a unicyclist. The actual system is shown below, as is an approximate model.



a) If the unicyclist is traveling at 3 *m/s* and hits the bump illustrated above, what will be the cyclist's maximum deflection from x_{eq}?

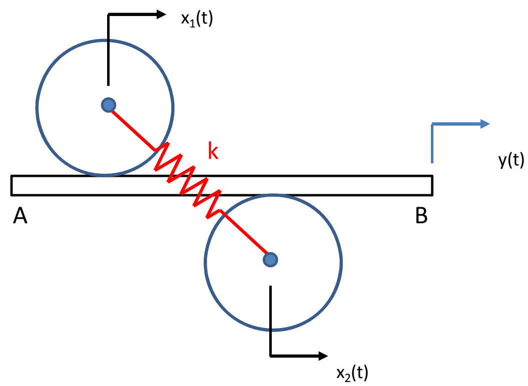
b) If the surface on which the unicyclist is traveling has a sinusoidal profile, as shown below, what is the worst speed for travel, and why?



c) Now, in parallel with the spring, add a damper with a damping coefficient c = 88 N/m/s to the unicycle. Under steady-state conditions, at what velocity will the maximum response occur?

Problem 2

Two identical disks of mass m, radius R, and mass moment of inertia $mR^2/2$, roll without slipping against track AB. A spring of stiffness k connects the two centers of the disks such that the spring is unstretched when the spring makes and angle of 45^O. Track AB is prescribed to oscillate harmonically as $y(t) = Y_0 \sin(\omega t)$.



a) Find the linearized equations of motion using the coordinates x_1 and x_2.
b) Find the natural frequencies and natural modes of the system.
c) Find the steady-state amplitude of response for x_1 and x_2 as functions of ω.

Problem 3

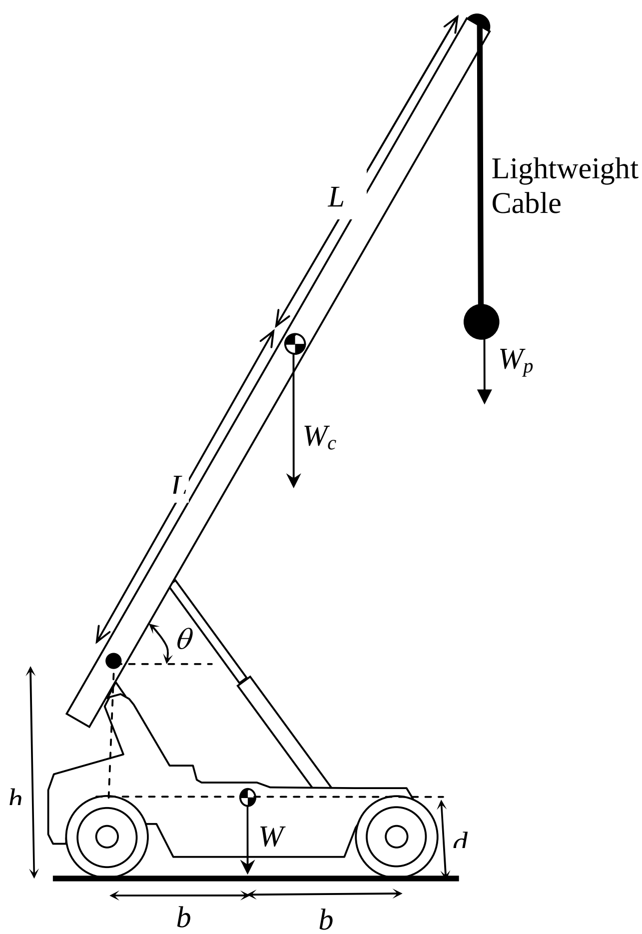
You are asked to analyze stability of the crane-equipped utility vehicle shown below. Throughout the problem you may assume the following relationships among the component weights and dimensions: $W=2W_c=2W_p$ and $L=2b=1.5h=4d$.

a) For what values of the boom angle θ is it possible for the vehicle to be in static equilibrium?

b) Assuming a fixed angle θ, determine the maximum allowable vehicle linear acceleration/deceleration from rest in terms of θ for a stable vehicle operation (i.e., no tipping over). Sketch the maximum allowable acceleration vs. θ.

c) Does your answer in (b) depend on whether the vehicle is front or rear wheel drive? Is there any advantage of one platform over the other to operate the vehicle at low accelerations on slippery surfaces? Explain.

Please clearly state all the assumptions you have to make as well as any additional variables that you may need to introduce.

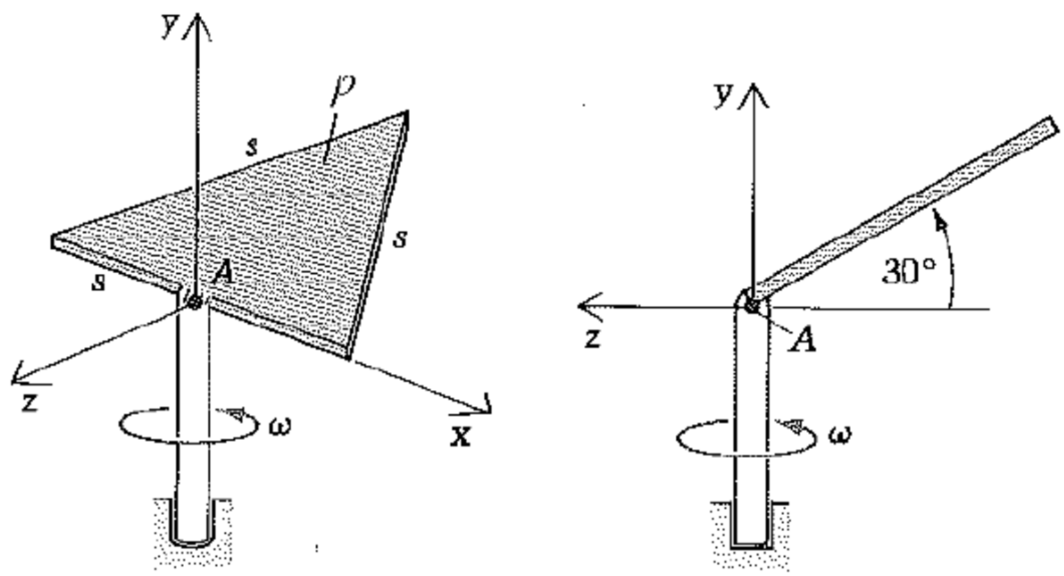


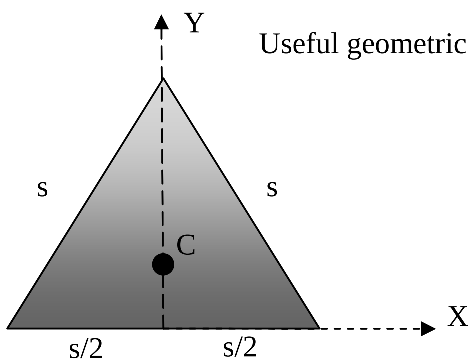
Problem 4

A thin equilateral triangular plate ℙ of mass m and sides s is welded to the vertical, massless, shaft at point A as shown in the figure. The shaft is brought up to a speed ω_0 from rest by a motor. In the questions below, **neglect gravity**.

a) How much work is done in bringing the system up to speed?

b) Find the reactions at A after the plate is brought up to speed ω_0 and the motor is turned off.





Useful geometric properties - X,Y,Z oriented

$I_{XX} = ms^2/8$
$I_{YY} = ms^2/24$
$I_{ZZ} = ms^2/6$

$Y_c = H/3$ where H is the height of the triangle