GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Fall Semester 2010

DYNAMICS & VIBRATIONS

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

✱ Please sign your **name** on the back of this page —

Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2010

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

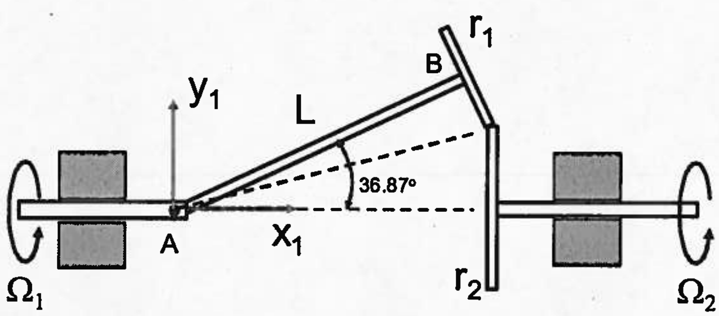
Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

Gear 1 of mass m and radius r_1 spins relative to its shaft AB, which is pinned at location A to an input shaft (shaft 1) having constant rotation rate Ω_1. Gear 1 rolls without slipping on Gear 2, which has radius r_2 and is driven at constant rotation rate Ω_2. The coordinate system $x_1y_1z_1$ is fixed to shaft 1 such that the frictionless pin at location A is aligned with the z_1-axis. You may assume that shaft AB is massless.

(a) For the case where Ω_2 is zero, find the maximum speed Ω_1 such that Gear 1 does not lose contact with Gear 2. Concentrate on finding the equation which, when solved, will yield this critical speed.

(b) Explain what effect Ω_2 has on your answer to part (a). Is the tendency for "lift-off" increased or decreased as Ω_2 takes on large positive or large negative (constant) values?



Problem 2.

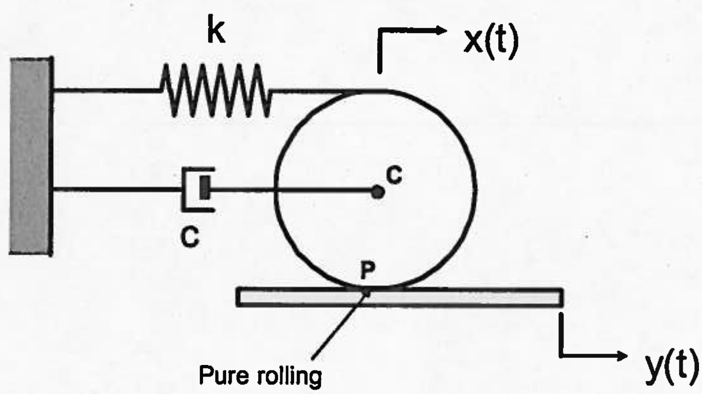
A uniform disk of radius r, mass m, and mass moment of inertia $I_C = mr^2/2$, rolls without slipping on a moving surface. The disk is connected to ground by means of a linear spring and damper as shown. The surface moves horizontally with a prescribed motion, $y(t)$; the displacement of the disk's center C is denoted x, and the spring and damper are undeformed when $x = y = 0$.

(a) Find the governing equation for this single-degree-of-freedom system.

(b) Assuming that $y(t) = Y_0 \cos(\omega t)$, find the amplitude of the response X as a function of the excitation frequency ω and other system parameters.

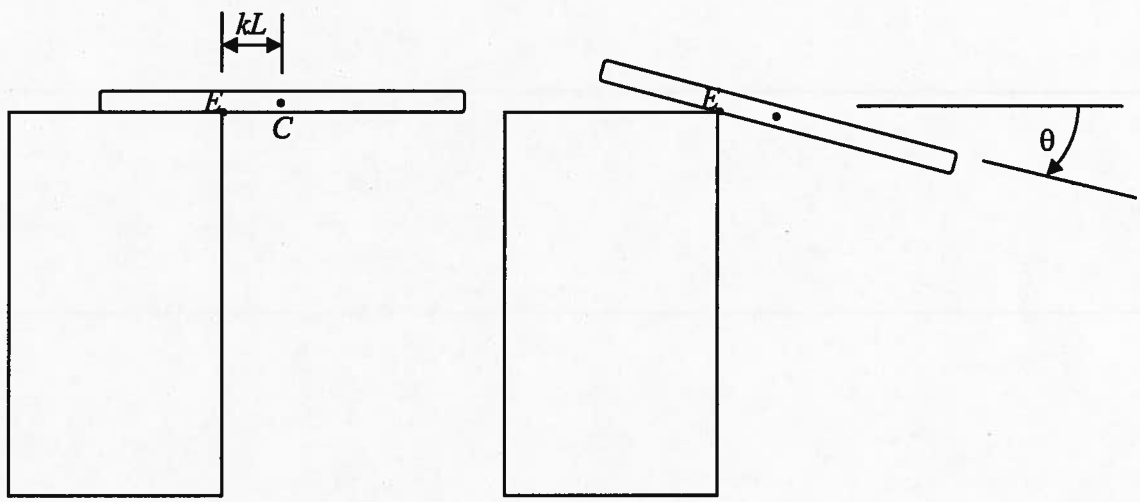
(c) Is there a frequency at which X is identically zero? If so, find the value of this frequency.

(d) Discuss how one would test the assumption of pure rolling, if the coefficient of friction was given as μ.



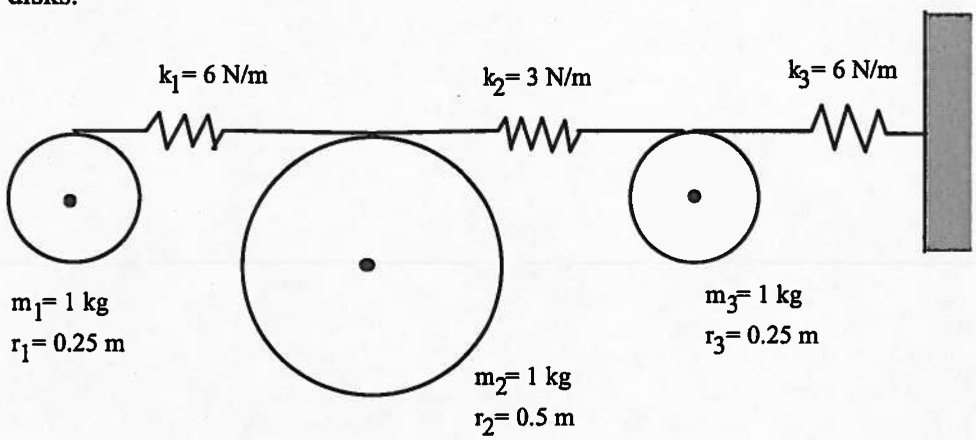
Problem 3.

A slender, uniform rod of length L is placed on a table as shown in the left sub-figure. It is placed such that the mass center C is located a distance KL away from the contact point E. Just after setting the rod in place, it begins to rotate about E, as depicted in the right sub-figure. Some time later a critical angle θ_s will be reached at which point the rod will begin to slide away from the table. Find this angle assuming the coefficient of friction between the rod and the table to be μ.



Problem 4.

Consider the system below. It is comprised of three thin uniform disks on massless, frictionless shafts, and interconnected at their rims by linear springs. The mass moment of inertia for a uniform disk is $\frac{1}{2}mr^2$. Assume the springs always act tangentially to the disks.



(a) Find the equation of motion for this system, and express in matrix form

(b) If the second natural frequency of this system is 4.103 rad/s, determine the corresponding mode shape.

(c) Determine whether or not $\underline{\phi}=\begin{Bmatrix}1\\0.427\\0.315\end{Bmatrix}$ and $\underline{\phi}=\begin{Bmatrix}1\\-0.726\\0.763\end{Bmatrix}$ are modes of the system.