GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2002

Dynamics and Vibrations
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Ph.D. Qualifying Exam – Dynamics and Vibrations

Fall, 2002

Instructions

Work any 3 of the 4 problems in this exam. It is important that you clearly mark which 3 problems you wish to have graded. (You may circle the problem numbers you wish to be scored, below).

For the 3 problems that you select, be sure to show all your work, assumptions, etc. in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave the algebra until the end. When necessary, you may leave your answers in unevaluated numerical expressions.

Good luck!

Circle which problems you wish to have scored:

1

2

3

4

PROBLEM #1

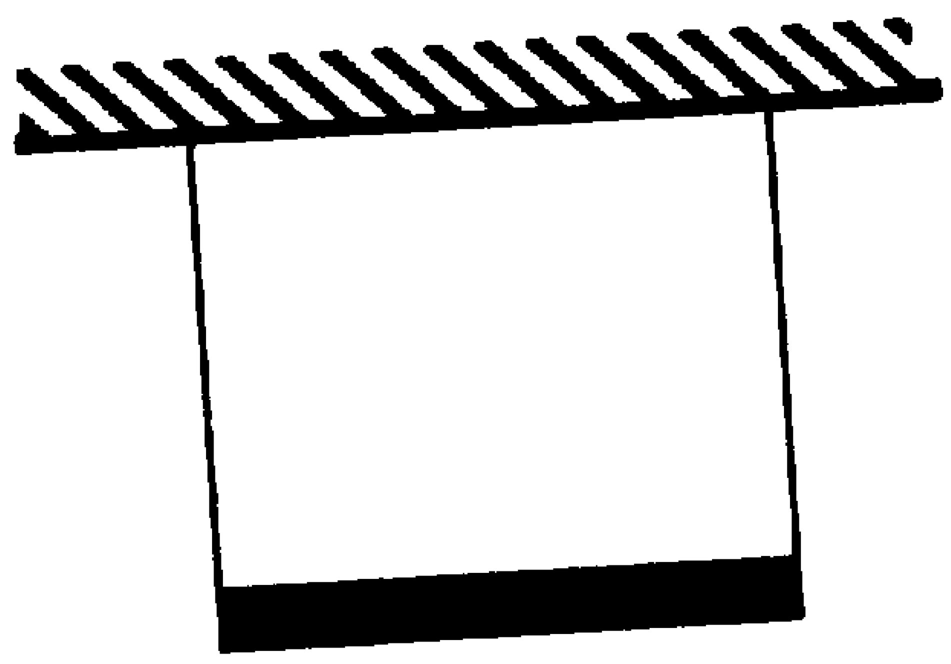
A homogeneous sphere of mass m and radius r rolls without slipping on a rough horizontal plane. The plane rotates with constant angular velocity Ω about a fixed vertical axis OZ. If the only external force on the sphere is gravity (and, of course, the plane's reaction on the sphere, acting at the latter's contact point with the plane, C) prove that the center of the sphere G (which is also its center of mass) traces , relative to inertial/fixed space, a circle whose center and radius depend on the initial conditions. The moment of inertia of the sphere about an axis through G equals: (2/5) (mass) (radius squared).

HINTS: Take inertial/fixed axes O-xyz, with O-xy parallel to the spinning plane and O-z positive downwards. Then, the coordinates of G will be $(x, y, 0)$ and the coordinates of C will be (x, y, r), while the equation of the spinning plane will be $z = r$.

PROBLEM #2

A uniform slender rod is only supported by wires attached to both ends. As a result, this rod is originally in a horizontal position. If the right-hand string is cut, find the initial tension in the left string. The slender rod has mass m and length L, while C denotes the center of mass.

$$\left(I_C = \frac{mL^2}{12}\right)$$



PROBLEM #3

Consider the following equation of motion:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

1) By inspection, will this system exhibit rigid-body motion? Justify your answer.

2) Sketch a spring-mass system that would yield the above equation of motion.

3) Determine the natural frequencies and mode shapes for this system.

PROBLEM #4

A time dependent displacement $y(t)=y_o cos(t)$, is applied at the free end of spring 2 of the system depicted in the figure, below. The two rods can tilt sideways in the plane of the paper about the pivot. There is some viscous damping at the pivot caused by bearing friction. An experimental natural response of the system is shown below. Based on the system geometry ($l_1=1\ m$, $l_2 = l_3 = 0.5\ m$) and stiffness parameters ($k_1 = k_2 = 220\ N/m$), and letting letting $y_o = 10\ mm$, determine, to the best of your ability:

1) the system mass
2) the magnitude of damping
3) the maximum angle (tilt) of the forced response
4) estimate at which ω the system response is maximum.

Assume the rods to be massless, and that gravity is acting downwards.