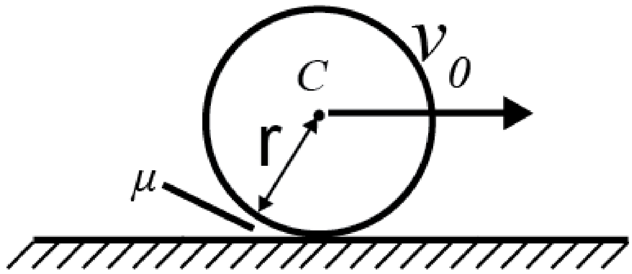
Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2017

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

A uniform sphere of mass m and radius r is projected along a rough horizontal surface with an initial linear velocity v_0 and no angular velocity, as shown. The sphere is vertically acted on by gravity. Let μ be the coefficient of kinetic friction between the sphere and the surface.



(a) Determine the time t_1 at which the sphere will start rolling without slipping.

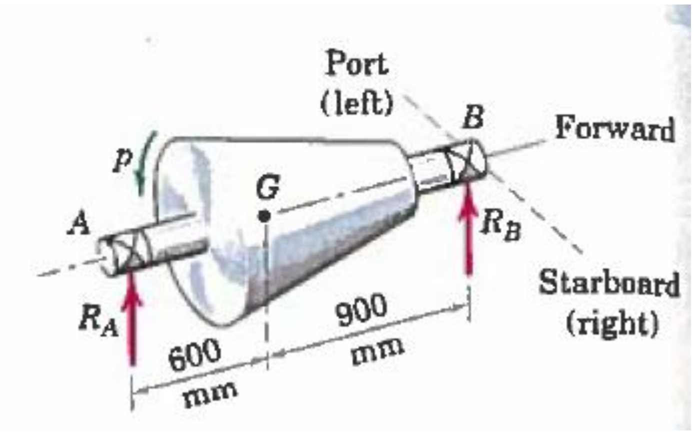
(b) Determine the linear velocity, v_1, and the angular velocity, ω_1, of the sphere at time t_1.

(c) Discuss how t_1, v_1, and ω_1 will change if the sphere were instead a cylinder. There is no need to complete a new analysis.

Problem 2.

The turbine rotor in a ship's power plant has a mass of 1000 kg, with center of mass *G* and a radius of gyration k_G of 200 mm about its spin axis. The rotor shaft is mounted in bearings *A* and *B* and turns counter clock-wise (as seen looking from *A* to *B*) at a speed of 500 rev/min. If the ship is making a turn to the left of 400 meter radius at a speed of 25 knots (1 knot = 0.514 m/s),

(a) Determine the vertical components of the bearing reactions at *A* and *B*.
(b) Will the front of the ship tend to rise or fall during this turn?



Problem 3.

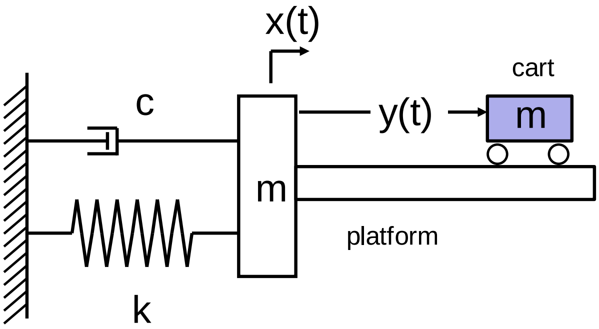
A platform of mass m is restrained to ground by a spring *k* and a viscous damper *c*. Engaged with the platform is a cart also having mass *m* that moves along the length of the platform. The relative displacement *y(t)* is specified to be $L + Y\sin(\omega t)$, where *L* is a constant ($L > Y$, thus no impact). Note that for *y(t)* to have the given motion, a force must be applied to the cart from the platform (e.g., via rolling resistance), and a horizontal reaction force must be exerted back onto the platform from the cart.

(a) Show that the equation of motion for *x(t)* is $2m\ddot{x} + c\dot{x} + kx = -m\ddot{y}$. Hint: in your FBD, account for the action-reaction between the cart and the platform by an unknown force.

(b) Assuming very small damping, what frequency should *y(t)* be given so as to produce the greatest amplitude of motion in *x(t)*? Express your answer in terms of the system parameters.

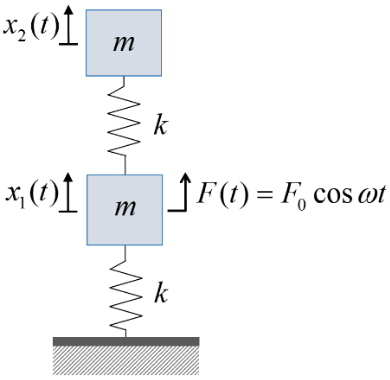
(c) Sketch the plot of the amplitude of the platform displacement *x(t)* vs frequency. On your sketch, show how this curve is influenced by the size of the damping ratio.

(d) As the frequency ω goes to infinity, show that the amplitude of the motion for *x(t)* approaches *Y*/2. Can you explain physically why that is the case?



Problem 4.

Consider the following undamped 2-DOF system. The displacements $x_1(t)$ and $x_2(t)$ are measured from the static equilibrium position shown below (no gravity).



(a) Express the forced equations of motion in matrix form.

(b) Analyze the system for free vibrations, i.e. $F(t) = 0$. Obtain the natural frequencies (in terms of *k* and *m*) and mode shapes.

(c) Derive the forced response (i.e. particular solution) $\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}$ to given harmonic excitation $F(t) = F_0 \cos \omega t$ by modal analysis or by matrix inversion.

(d) Let *m* = 10 kg, *k* = 1000 N/m, and $F_0 = 1$ N. Calculate the excitation frequency ω at which the forced response $x_1(t) = 0$. What is the forced response $x_2(t)$ at that frequency?