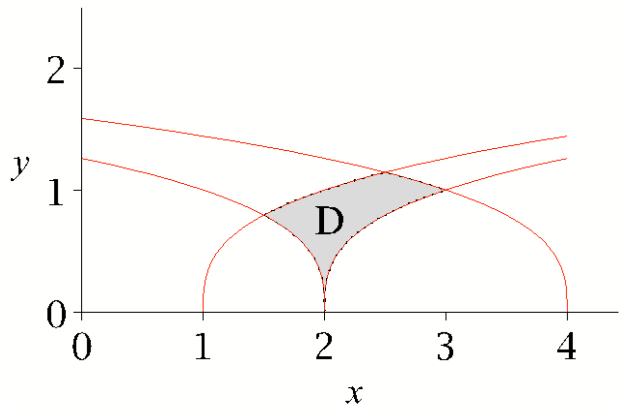
APPLIED MATH WRITTEN EXAM

Please solve all 4 questions

1. Let D be the planar domain bounded by the curves $x + y^2 = 2$, $x + y^3 = 4$, $x - y^3 = 1$, and $x - y^3 = 2$. Evaluate $\iint_D xy^2 + y^5 \, dA$



2. Develop a 4^{th} order polynomial to curve fit the following 5 given data points of:

x	f(x)
3.2	22.0
2.7	17.8
1.0	14.2
4.8	38.2
5.6	51.7

3. Imagine a launching system, developed to make a projectile escape the earth's gravity. The launching system is buried in the earth. When the projectile leaves the launcher, exactly at ground level, its speed should be v_0 (= escape speed) in order to escape the earth's gravity.
The projectile within the launcher undergoes an acceleration from zero speed to speed v, where we demand (by tuning the system) this speed v to be exactly the escape speed v_0. (NOTE THAT YOU CAN PROCEED TO PART B EVEN IF YOU CANNOT FIND v_0 IN PART a)

a) Determine v_0, i.e. the velocity required to escape the earth's gravity, where the weight of an object (the gravity force acting on it) is given by

$$w(x) = P/(R + x)^2$$

P being a parameter you can determine based on the gravity expression at ground zero and x being the height above ground zero, where R is the earth's radius. We ignore effects caused by the atmosphere.

b) Once you know v_0, solve the following problem. The launching system's behavior is described by the differential equation, containing adjustable parameters (variables) y and z:

$$2y + z^2 + 2yzz' = 0$$

where z is a function of y and where, in the case the control system adjusts the parameter y to exactly v_0 , z is, in this case, exactly equal to 0. Determine the algebraic equation, governing the control system, that contains no derivatives, and that describes how z and y are linked to one another.

4. For $A = \begin{bmatrix} 0 & -2 & 1 \\ -2 & 3 & -2 \\ 1 & -2 & 0 \end{bmatrix}$,

a) Determine the eigenvalues for *A*.

b) Determine an orthonormal basis for $\mathbb{R}^3$ that are eigenvectors for A.

c) Find an orthogonal matrix Q and diagonal matrix D such that AQ = QD.