APPLIED MATH WRITTEN EXAM

FALL 2019

1) Answer the following questions:

(a) Consider the linear algebra problem defined by $Ax = b$, where A is a symmetric non-singular matrix. If v is an eigenvector of the matrix A and v is perpendicular to the right-hand-side vector b, show that v must also be perpendicular to the solution vector x.

(b) Consider the linear algebra problem defined by $Ax = b$, where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$.

Solve this problem for the solution vector x using an *eigenvector expansion* and demonstrate that this problem satisfies the conditions of part (a).

2) Answer the following questions:

(a) Evaluate the flux of the vector field $\vec{v} = \eta\,\hat{i} + \beta z\hat{k}$ **through the surface enclosing the volume of a cylindrical region bounded by** $1 \le x^2 + y^2 \le 2$ **and** $0 \le z \le 2$**, where** η **and** β **are constants.**

(b) Prove that $\text{div}\left(\text{curl}\,\vec{A}\right) = \vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$**, where** $\vec{A}$ **is a vector.**

(c) Let $f(x, y) = e^{xy}\cos\left(x + 2y\right)$**. At the point** $\left(0, \pi/2\right)$**, in what direction does the function f change most rapidly?**

3) Consider the following equation with y(x):

$$\frac{d^3y}{dx^3} - 3\frac{dy}{dx} - 2y = 3e^{-x}$$

a) **Find the general solution of the homogeneous equation**
b) **Find a particular solution of the entire equation, which does not have undetermined coefficients**
c) **Provide a general solution for the entire equation**

4) Solve the following set of equations using a Newton-Raphson iterative method. Show the work for two iterations.

$$2x^2 + y^2 - 8 = 0$$

$$x^2 - y^2 + xy - 4 = 0$$