APPLIED MATH WRITTEN EXAM

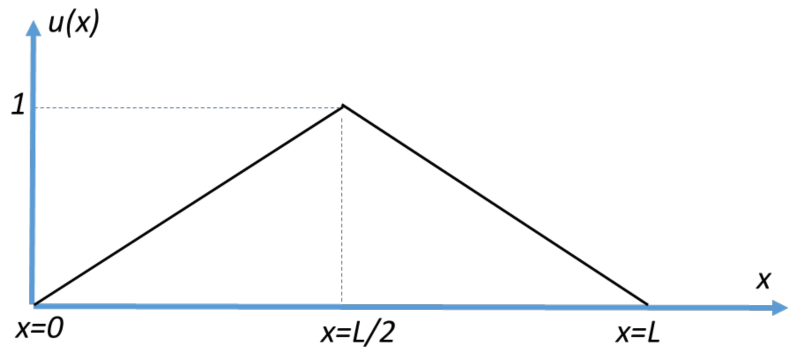
Please solve all 4 questions

1. Use Green's Theorem to evaluate $\oint_C \, [(x - xy)dx + (y^3 + 1)dy]$ where *C* is the boundary of a square with vertices *(x,y)* at the points at (1,0), (2,0), (2,1), and (1,1).

2. String vibration is described by the following partial differential equation in the limit of a small displacements ***u*** (where ***x*** is a coordinate along the string length, ***t*** is time, ***a*** is a wave velocity which depends on the string tension and linear density and is assumed to be known).

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$$

Complete the formulation of the problem by specifying the boundary/initial conditions for vibration of a string with two fixed ends (at ***x=0*** and ***x=L***), which is initially at rest and pulled at a mid-point (see Figure below) and then released at ***t>0***. Classify the governing equation based on its mathematical type and derive an analytical solution of the problem for displacement ***u*** as function of ***x*** and ***t***.



3. Determine the coefficients “a” and “b” in the expression of:

$$y = a\,(\ln x) + b\,(\cos x)$$

such that the expression can best represent, in the sense of least-square-error, the following given data

x	1	2	3	4
y	0.23	-0.26	-1.10	0.43

4. Imagine a stage at a theatre. One of the tasks of the director is to construct a moving object (e.g. a plastic bird) onto which 2 spotlights and one video camera should be targeting. The idea is of course to follow the object with the camera and simultaneously to have sufficient light shining on the object. Each device (camera or spotlight) is setup at a different location and has certain (limited) mechanical degrees of freedom (position on a rail for instance, and angle) to aim along a trajectory (mathematically spoken this would be a line in 3 dimensions) that corresponds to a linear equation in the three spatial coordinates x, y and z.

These are the equations:

Equation 1 (camera):

$$x + 2y + 3z = c$$

Equation 2 (spotlight A):

$$3x + 4y + 6z = c - 3$$

Equation 3 (spotlight B):

$$-4x + 6y + 9z = 2$$

a. Because the idea is to aim the 3 devices at the same spot, determine the value of c, and therefore determine the constraint on the system by the requirement that they should all aim at the same spot simultaneously, in other words the 3 mathematical lines should intersect.

b. Once you know the value of c, assume that the given object can move around in 3D space while it is continuously aimed at by the 2 spotlights and by the camera. To make this happen the object can only follow a path where the two spotlights and the camera can simultaneously aim at. Determine the mathematical equation of this path.

c. Where and at what angle may the moving object (e.g. the plastic bird) impact the stage when the stage (i.e. a plane surface) is described by the coordinates $(x, y, 0)$.

d. One of the actors on stage is located at $(x, y, z) = (0,0,1)$, how close can the moving object (plastic bird) possibly get to that actor?