Dynamics Systems & Control Ph.D. Qualifying Exam
Spring 2015

<u>Instructions</u>:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1

Figure 1(a) shows an axle-box bearing test rig, where an actuator (piston area a) applies the simulated axial-force at the end of the main shaft. Because of large axial load, the clearance (modeled as stiffness k_s) between the inner and outer bearing rings along the axial direction may not be neglected. The dynamic effect of the clearance compliance can be accounted for by the constitutive relationship graphically illustrated in Fig. 1(b) where k_h and k_s are the high and soft stiffness; $\pm d$ define the boundaries of the bearing clearance; and thick and dashed lines are the force-displacement relationships with and without considering the clearance compliance respectively. In the actual test rig, measures are taken to ensure that the friction $f_s \approx 0$ and the leakage $q_l \approx 0$. The dynamics of the hydraulic actuator can be characterized by

$$G_v(s) = \frac{\Delta P(s)}{U(s)} = \frac{k_v \omega_v^2}{s^2 + 2\zeta_v \omega_v s + \omega_v^2} \quad \text{where } \Delta P = P_2 - P_1.$$

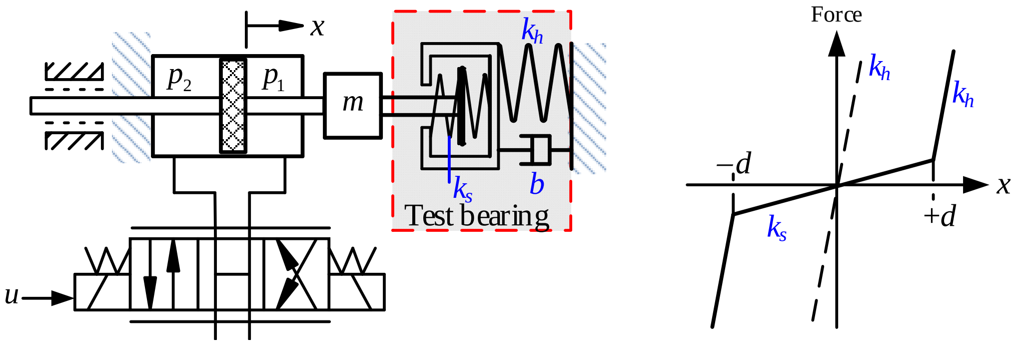


Fig. 1 Schematic of axial loading system

a) Derive the transfer function relating the input $u(t)$ and output $x(t)$ for the system without considering the clearance.

b) Give the equations for modeling the clearance.

c) Discuss the clearance effect on the step response of the piston motion $x(t)$; you may neglect dynamics of the hydraulic actuator in this discussion.

Problem 2

It is interesting to have dynamical interpretations of zeros and poles of a system. Consider a dynamic system whose input-output transfer function is

$$G(s) = \frac{s+1}{s^2+5s+6}.$$

(1) What are the zero(s) and poles of the system?

(2) Let $u(t)$ and $y(t)$ denote the input and output, respectively. Show that there exist an initial condition of the system and a constant v such that if $u(t) = \exp(vt)u_s(t)$, then $y(t) = 0$ for all $t \geq 0$, where $u_s(t)$ denotes the unit-step function. What are the initial condition and v?

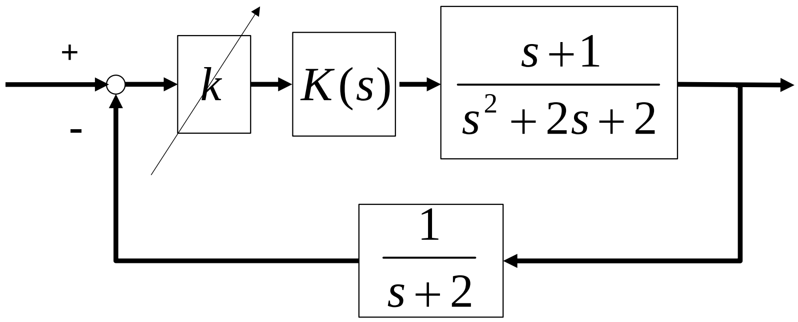
(*The result sounds like you keep on pumping energy into the system and don't get anything out!*)

(3) Show that there exist an initial condition of the system and a constant v such that if $u(t) \equiv 0$, then $y(t) = \alpha \exp(vt)u_s(t)$, where α is a constant. What are the initial condition and v?

(*The result sounds like you get something out of the system for free!*)

Problem 3

Consider the following feedback system with a loop gain *k* (>0).



a) Assume controller $K(s) = 1/s$ (i.e., integral control).

   (a-1) Sketch the root-locus and discuss the stability of the feedback system with respect to *k*.

   a. (a-2) Determine the range of *k* for stability.

b) Give an example of $K(s)$ such that the feedback system is stable for any *k* >0. The answer is not unique. Controller *K*(s) must be realizable. Justify your design.

c) Note: When sketching a root-locus plot, you don't need to determine the angle of departure from complex poles, intersection of the asymptotes, or break-in/away points. A general sketch is acceptable.

Problem 4

The input-output relationship of a dynamic system is given by $T\dot{y} + y(t-d) = u(t-d)$ where T and d are positive constants.

a) Find the input-output transfer function $G(s)=Y(s)/U(s)$.

b) For what values of T and d is $G(s)$ stable? *Hint*: You may express the characteristic equation of the system as $1+H(s)=0$ and use frequency response based criteria to derive the stability condition.

c) Derive an expression for the steady-state response of the system to $u(t)= a \sin\omega t$ where a and ω are positive constants. Is your answer valid for all values of T and d? Explain.