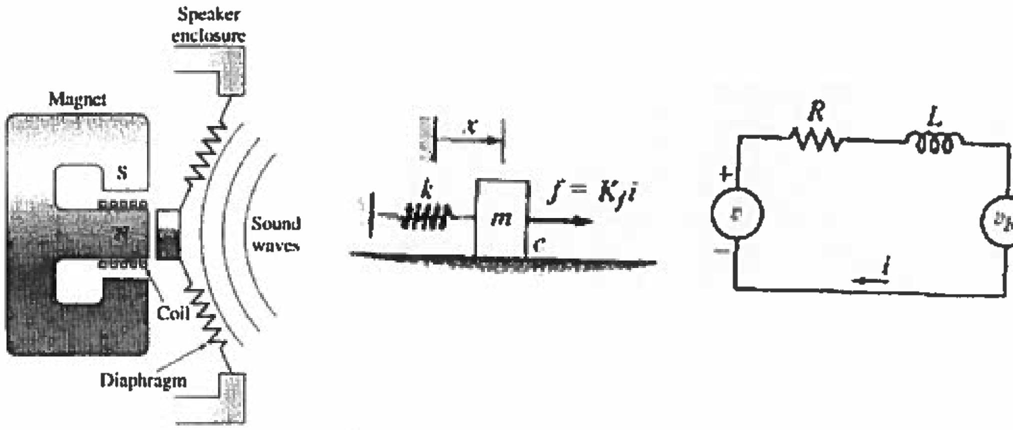
SDC PhD Qualifying Exam
Spring 2014

Work all 4 problems

Problem 1.

The figure shows a speaker and its electromechanical model. The mechanical model has mass m, damping c, and stiffness k; x is the displacement and f is the force applied by the electromagnet. The electromagnet is modelled by resistance R, inductance L, back emf V_b, current i and applied voltage v. The force generated by the electromagnet is proportional to the current, $f = K_f i$.

a) Determine the transfer function X(s)/V(s)
b) It is desired to represent the transfer function in the following form, $X(s)/V(s) = G_1G_2/(1+ G_1G_2H)$ where the plant G_2 represents the mass.
   i) Draw the block diagram identifying G_1, G_2, and H
   ii) What type of control does G_1 represent?



Problem 2.

Consider a unity-feedback system whose open-loop (feedforward) transfer function is

$$G(s) = \frac{s-1}{s(s-1)}.$$

(1) Find the closed-loop transfer function. Notice that this is a second-order system.

(2) By finding the poles, determine if the closed-loop system is stable or unstable.

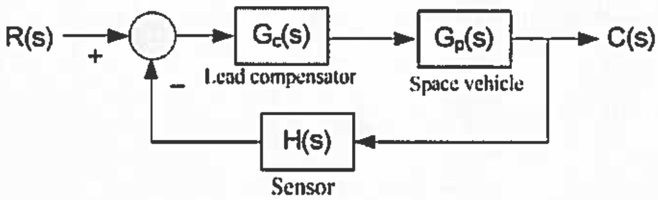
(3) Suppose the input is a unit-step function and the initial conditions are zero. Let $y(t)$ denote the output of the closed-loop system. Find $y(t)$, $t \geq 0$. Find also $\lim_{t\to\infty} y(t)$. Is this result consistent with your answer in (2)? Explain.

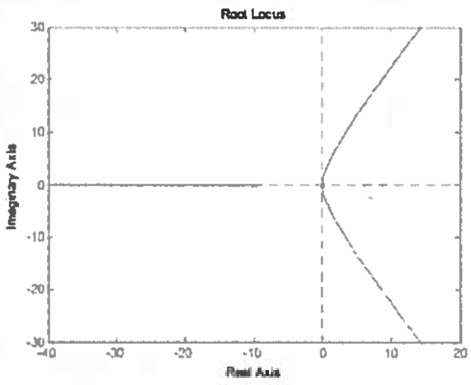
(4) Suppose the input is zero. What is $y(t)$, $t \geq 0$, if the initial conditions are non-zero? Find also $\lim_{t\to\infty} y(t)$. Is this result consistent with your answer in (2)? Explain.

(5) Suppose the input is zero. Is it possible to select a particular set of initial conditions which render $\lim_{t\to\infty} y(t) = 0$? Explain. Is this result consistent with your answer in (2)? Explain.

Problem 3.

Consider the model for a space vehicle control system shown below, where $G_p(s) = 1/s^2$. Design a lead compensator $G_c(s)$ such that the damping ratio and un-damped natural frequency of the dominant closed-loop poles are 0.5 and 2 rad/s, respectively. Given the root locus for the uncompensated system $G_c(s) = 1$, an engineer suggests choosing the zero of the compensator at $s = -1$ as an initial guess. Find the corresponding gain and pole of the compensator to meet the specification. Sketch the root locus of the compensated system and discuss briefly the effect of the zero at $s = -1$.





Problem 4.

Problem: Consider the following phase-lead compensator:

$$C(s) = \frac{s + T_1}{s + T_2} \quad (T_1 > 0, T_2 > 0)$$

Determine T_1 and T_2 such that the maximum phase lead created by $C(s)$ is 45 [deg] at 1 rad/sec. Use $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$ if necessary.

$C(j\omega)$