Dynamics Systems & Control Ph.D. Qualifying Exam
Fall 2014

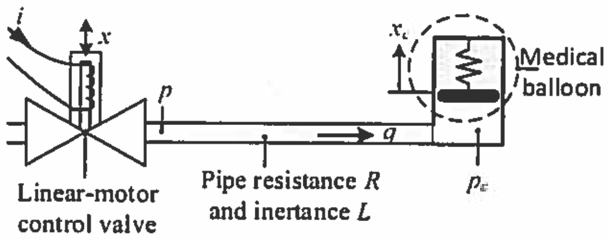
Instructions:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.
Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

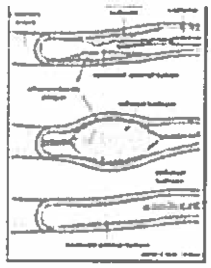
Problem 1

Fig. 1(a) shows a schematics for a medical application (Fig. 1b) where liquid at supply pressure p_s flows through a variable valve to a medical balloon (under pressure p_c) to widen an artery for clearing a blockage. As illustrated in Fig. 1 where all variables are measured from an operating point $(\bar{x}, \bar{i}, \bar{p}_s, \bar{p}, \bar{p}_c, \bar{q}, \bar{f}_m)$, the balloon (along with its surrounding effect) is modeled as a mass-less piston (with area A) against a spring-damper (k_c and b_c) system. The pressure difference across the valve can be approximated by $p_s - p = c_q q - c_x x$. The valve is actuated by a linear-motor that generates an electromagnetic force linearly proportional to the product of the displacement x and controlling current i. With negligible damping and friction, the motor armature (mass m) is held against a calibrated spring with its stiffness adjusted to $k = a_x$.

Show that the system can be represented by the block diagram shown in Fig. 2. Derive the transfer function that relates the balloon pressure $P_c(s)$ and cntrolling current $I(s)$ when the supply pressure is held at $\bar{p}_s$.



(a)



(b)

Fig. 1

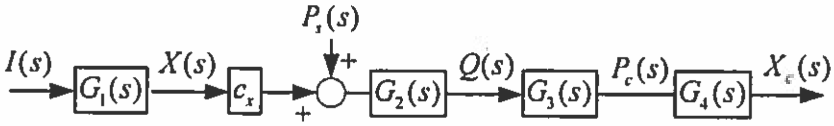


Fig. 2

Problem 2

Consider a unity-feedback system whose open-loop transfer function is

$$G(s) = \frac{s+1}{s^2+3s+5}$$

(1) What is the closed-loop transfer function?
(2) Explain the meaning of stability and instability.
(3) Is the closed-loop system stable or unstable? Why?
(4) Suppose the output of the system is $y(t)$, whose initial conditions are $y(0_-) = 1$, $\dot{y}(0_-) = -1$. Suppose the input is $u(t) = u_s(t)$, where $u_s(t)$ stands for a unit-step function. Find $y(t)$, $t \geq 0$.
(5) Discuss the consistency between (2) and (4).

Consider instead a unity-feedback system whose open-loop transfer function is

$$G(s) = \frac{s+1}{s^2-3s+5}$$

(6) What is the closed-loop transfer function?
(7) Is the closed-loop system stable or unstable? Why?
(8) Suppose the initial conditions of $y(t)$ are $y(0_-) = 0$, $\dot{y}(0_-) = -1$. Suppose the input $u(t) = \exp(-t)u_s(t)$. Find $y(t)$, $t \geq 0$.
(9) Discuss the consistency between (2) and (8).

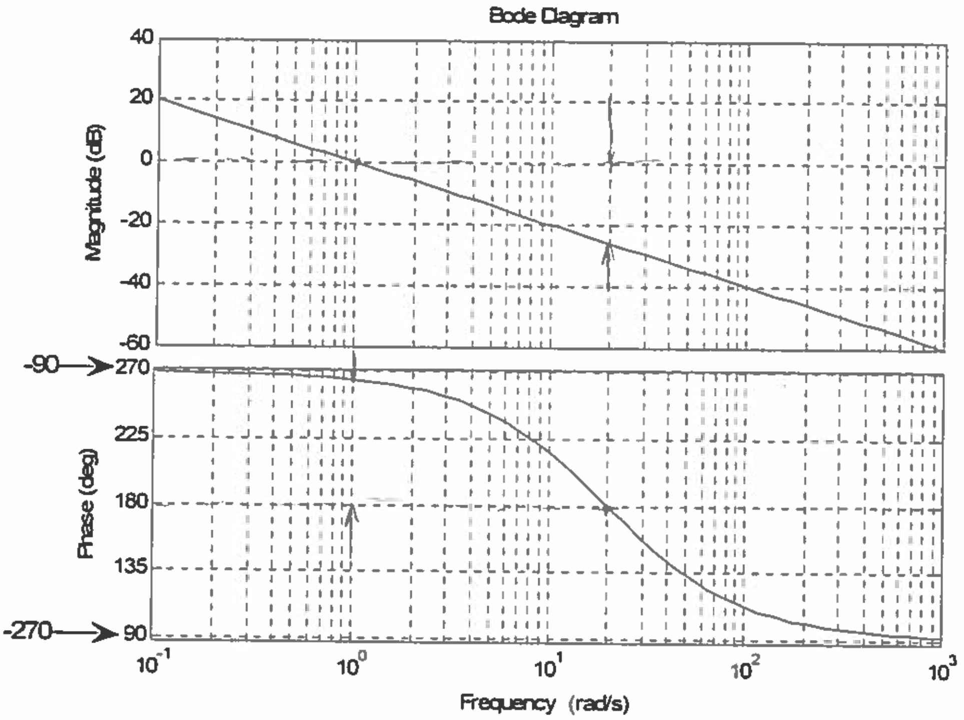
Problem 3

Consider the unity feedback of $G(s) = \dfrac{s+3.5}{(s-1)(s+3)}$ with a scalar feedback gain k (>0).

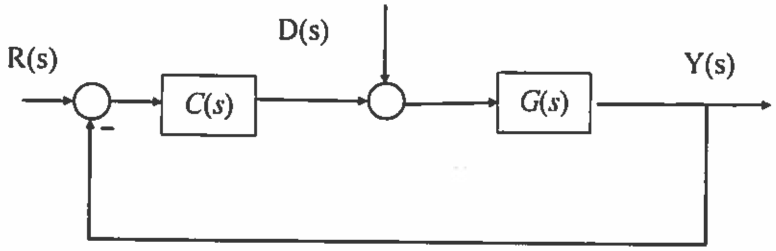
(a) Sketch the root-locus plot. Determine break-in/away points if they exist.
(b) Determine the range of gain k for stability.
(c) Determine the range of k such that the closed-loop system is underdamped.
(d) Determine k such that the closed-loop system is the least damped.
(e) Consider the range of k obtained in (c). Find the range of the final value of the unit-step response of the closed-loop system.

Problem 4

The bode diagram of a linear time-invariant system with transfer function G(s) is shown below:



You are asked to design a feedback controller C(s) (see the block diagram below) for this system to meet the design requirements specified below.



a) Identify G(s) from the bode diagram.

b) Design a controller of your choice (e.g., P,PD,PI,PID,Lead,Lag,...) such that the compensated system has a phase margin (PM) of at least 45 degrees, a positive gain margin (GM), and a gain cross-over frequency $\omega_c \geq 20$ rad/sec. (i.e., the frequency at which $|CG(j\omega_c)|=1$). Sketch the bode diagram of CG.

c) Find the reference input $r(t)$ of your controller in (a) that produces $y(t)=\sin 10t$ at stead-state assuming $d=1$.