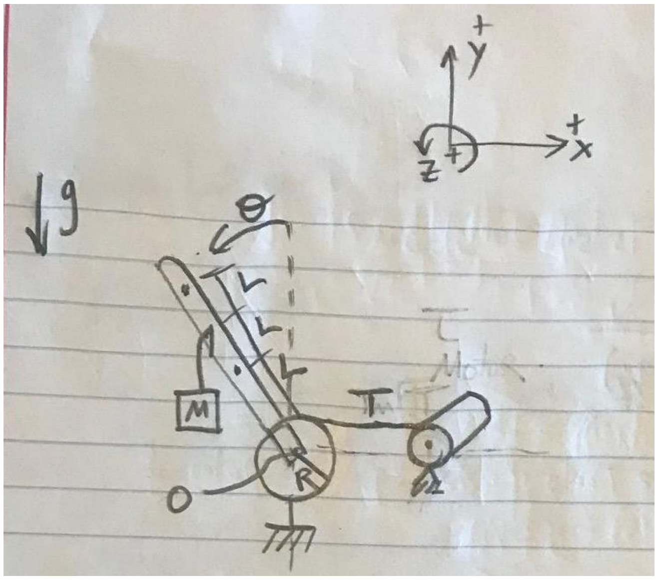
Dynamics Systems & Control Ph.D. Qualifying Exam
Spring 2019

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the 3 problems that you select, show all your work in order to receive proper credit. You are allowed to use a scientific, non-graphing calculator. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1: Consider the mechanical system shown below and answer the questions.



There is a massless arm attached to a pulley that is fixed to the ground. A cable attaches the pulley to an actuator. There is tension T in the cable. The pulley is massless, fixed to the ground and has center location O and radius R. The arm is massless but has an attached mass, M, which is placed in a position at length n*L (n=1, 2, 3..) from the center of the pulley. Gravity acts. θ is the counterclockwise + angle of the arm from vertical.

1) Obtain the transfer function from input tension T to output angular displacement θ or $P(s) = \frac{\theta(s)}{T(s)}$. Assume θ is small. Assume moment of inertia of the mass, M about point O is $M(nL)^2$.

2) Discuss the stability of $P(s)$. (1 sentence is OK)

3) Assume that the cable tension T is provided by an actuator according to:

$$T=K(x-x_0) + B(dx/dt)+T_c$$

with x_0 defined as the length of the cable when the arm is at top dead center i.e. θ=0 and x is the amount of cable unfurled from rest. T_c is a commanded tension.

Obtain a new transfer function from input tension T_c to output angular displacement θ or $G(s) = \frac{\theta(s)}{Tc(s)}$. Assume θ is small.

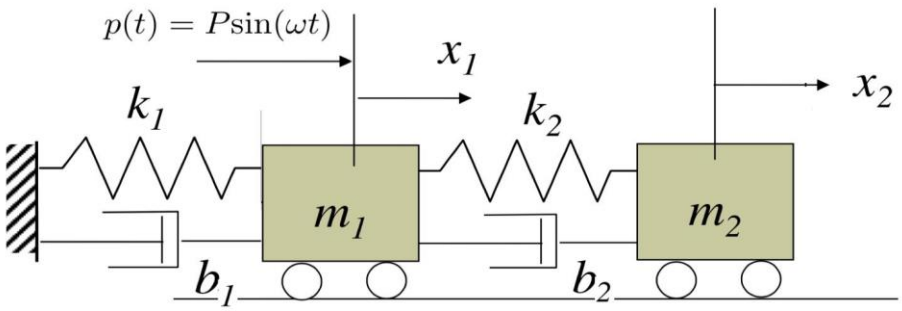
4) Discuss the stability of $G(s)$ (1 sentence is OK).

5) Assume M, R, K, B and L =1. Use the result in 4) and sketch the unit <u>step</u> response of the system output θ versus time for n=1, 2, 3. <u>Note:</u> A general sketch is acceptable where complete mathematical derivations using the inverse Laplace transformation are NOT required. However you should a) determine *the final value of the output*, and b) provide sufficient discussion or justification about *whether the response exhibits an overshoot or not*.

6) Solve for the value of n (i.e. the position of the mass along the arm) where it will be critically damped.

Problem 2: Consider the mechanical system shown in the figure. Displacements $x_1(t)$ and $x_2(t)$ denote the positions of mass 1 and 2. A sinusoidal input force $p(t)$ is exerted on mass m_1. Assume that the viscous damping coefficients b_1 and b_2 are positive. but negligibly small. [Hint: this means that, in obtaining equations, we may assume that $b_1 = 0$ and $b_2 = 0$. Since b_1 and b_2 are positive, however small, the system is stable and the following equation can be used to find the steady-state solution.]

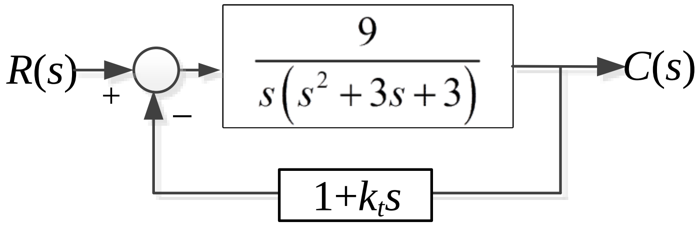
$$x(t) = |G(j\omega)|Psin(\omega t + \phi) = Xsin(\omega t + \phi)$$



where $X = |G(j\omega)|P$ and $\phi = \angle G(j\omega)$. $G(j\omega)$ is the sinusoidal transfer function with input P and output X. The displacements x_1 and x_2 are measured from the respective equilibrium positions in the absence of the excitation force. Assume that the ground is frictionless.

1) Derive the transfer function $X_2(s)/X_1(s)$ and $X_1(s)/P(s)$.
2) Find the steady-state displacements $x_1(t)$ and $x_2(t)$.
3) Determine whether the masses m_1 and m_2 move in the same or opposite direction. Please explain briefly. (Hint: judge the relationship between ω and $\sqrt{k_2/m_2}$ values)

Problem 3: Consider the feedback control system below:



1) Show that the characteristic equation for root locus analyses can be written as

$$1 + KG(s) = 0 \text{ where } G(s) = \frac{s}{(s+a)(s^2+b)}.$$

Determine the values of "a" and "b" and "K" in terms of k_t.

2) Sketch the root locus; specifically calculate the asymptotes (centroid and angles) and the departure angle at the complex root $s = j\sqrt{b}$.

3) An engineer claims that when K=3, the dominant closed-loop poles are located at $s = -0.42 \pm j2$; and the effect of the third pole on the transient response is negligible. Justify his claim by determining the third pole. Obtain an approximate second-order system for the closed-loop system; be sure that both the reduced and original models yield the same steady-state value.

Problem 4: The magnitude plot of the frequency response depicted in the following figure refers to the open-loop system of a unity-feedback control system.

1) For A=10, find the open-loop transfer function G(s) of the system

2) Write down the magnitude and phase equations with respect to the frequency, and roughly sketch the Bode diagram to determine the gain K value for the phase margin 45 deg.

3) Find the closed-loop transfer function F(s) of the system and compute the values of the damping ratio and natural frequency.