PLEASE NOTE: Answer 3 out of the 4 problems. In case you answer the 4 problems, clearly state which 3 problems you want to be graded.

## Problem 1

A circular section rod has a diameter of 20 mm and the shape of a quadrant of a circle of radius 300 mm , as shown in the figure. The Young's modulus of the material is 208 GPa and the shear modulus is 80 GPa . If a vertical force of 100 N is applied at the free end, determine
(1) The deflection of the free end in the direction of the force; and
(2) The stress state at point A which is on the surface of the rod, in the $x-y$ plane, and has a $y$-coordinate of $5 \sqrt{2} \mathrm{~mm}$.


## Problem 2

A cantilever beam of length $L$ is clamped at its left end, as shown below. The beam is initially straight, horizontal, and stress free. Underneath the beam there is a cylindrical rigid surface of radius $R$. The top of the cylindrical surface is initially in contact with the left end of the beam. A vertical force, $P$, is applied to the right end of the beam, causing a deflection of $\delta$ at the right end. The beam has a rectangular cross-section of width $b$ and height $h$, and is made of an elastic material with a Young's modulus of $E$ and a Poisson's ratio of $v$. Assume $R \gg L$.
(a). Qualitatively describe the deformation behaviors of the beam when $P$ is increased from zero to infinity.
(b). Derive a quantitative relationship between $P$ and $\delta$.


## Problem 3

Consider one loading to produce normal stresses on the x -direction of $\sigma_{\mathrm{x}}=80 \mathrm{MPa}$ and $\sigma_{y}=-30 \mathrm{MPa}$ in a brittle material. An additional shear stress is applied. This shear stress increases until a crack is observed to form at an angle of $\alpha=20^{\circ}$ (see figure below).
a) Find shear stress $\tau_{\mathrm{xy}}$ at which cracking occurs.
b) If this material were loaded in uniaxial tension at what critical stress would fracture occur?


## Problem 4

Consider the cylindrical structure shown made of a rod of Material A perfectly adhered to a shell of Material $B$. Both materials are perfectly plastic Material $A$ has modulus $E_{A}$ and cross sectional area $A_{A}$ and Material $B$ has modulus $E_{B}$ and cross sectional area $A_{B}$. The structure has length $L_{o}$.
A. What is the weight $W$ required to deform this structure to the strain $\varepsilon^{*}$, as shown on the Figure?
B. What is the length of the structure after $W$ is removed?
C. What are the residual stresses in Material $A$ and Material $B$ after $W$ is removed?


