PLEASE NOTE:
You need to correctly answer only 3 out of the 4 problems to receive full credit. In case you attempt all 4 problems, clearly state which 3 problems you want to be graded. If you do not explicitly so indicate, the 3 lowest scores will be used.

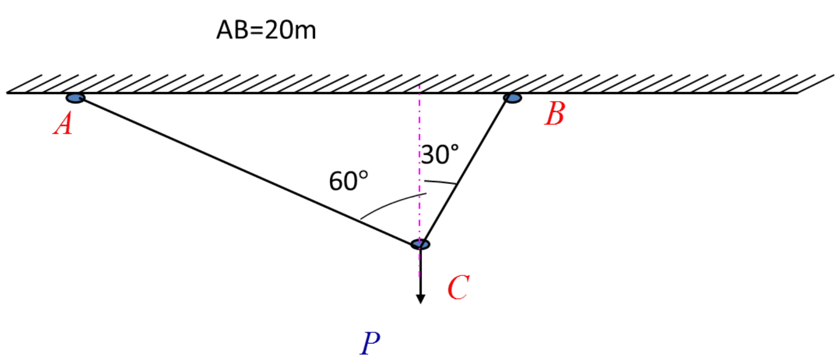
Problem #1

A load *P* is applied vertically downward at joint *C* of the simple truss *ABC* shown in the figure. Members *BC* and *AC* are made of steel and copper, respectively. The yield strength of steel and copper is 250 MPa and 110 MPa, respectively. The Young's modulus of steel is 200 GPa. The Young's modulus of copper is 130 GPa. Both *AC* and *BC* have a circular cross section with a diameter of 100 mm. Weight of the truss is negligible compared to the load. The moment of inertia of the circular cross section is $I_p=\pi d^4/64$.

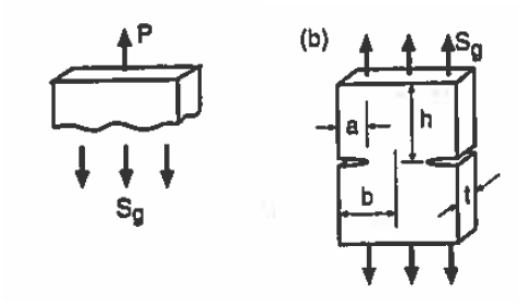
a) Assuming no failure at the joints and a factor of safety of 1.2, determine the allowable load of the truss.

b) Qualitatively describe the translation of point *C* from the original position when the allowable load is applied and explain why. Provide calculations and draw diagrams if necessary. Small angle approximation is allowed.

c) If the load at *C* is upward, determine the critical load.



Problem # 2:



Values for small a/b and limits for 10% accuracy:

$$\text{(b)}\quad K = 1.12 S_g \sqrt{\pi a}$$

$$(a/b \leq 0.6)$$

A double-edge-crack plate made of RQC-100 steel has dimensions, as defined in the figure above, of b = 75 mm and t = 6 mm. The two edge cracks have equal lengths of a = 0.3 mm, and the member is subjected to cyclic loading between P_{min} = -90 kN and P_{max} = 360 kN.

a) Estimate the number of cycles necessary to grow the crack to failure.
b) Repeat the question with P_{min} = 90 kN and P_{max} =360 kN.

Clearly list all your assumptions.

The Paris regime follows the equation $\dfrac{da}{dN} = C\Delta K^m,\ C = C_0(1-R)^{m(\gamma-1)}$

With R load ratio ($R = \sigma_{min}/\sigma_{max}$)

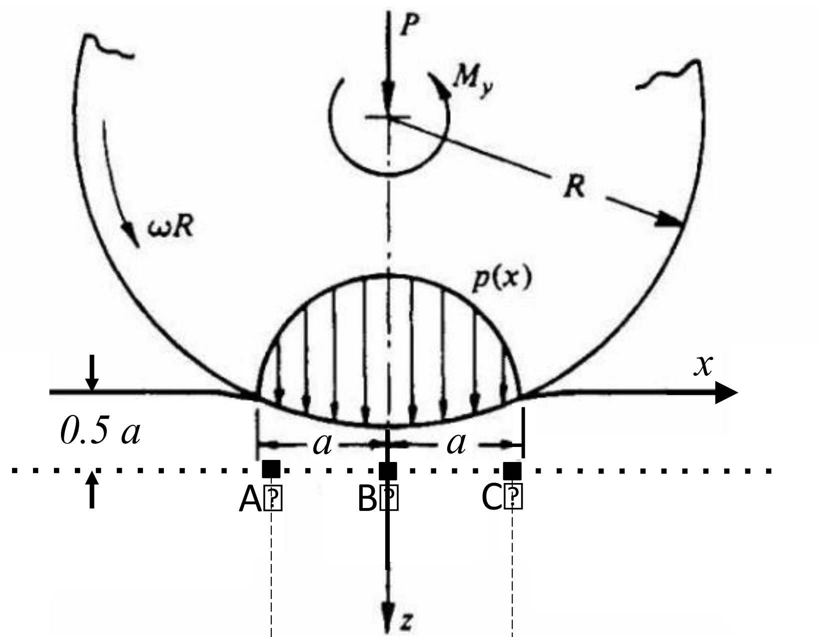
Mechanical properties of the material:

σ_o	K_{Ic}	C_0	m	γ	γ
MPa	MPa $\sqrt{\text{m}}$	$\dfrac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^m}$		(R ≥ 0)	(R < 0)
778	150	8.010E-14	4.24	0.719	0

Problem #3

The schematic represents the contact profile of an elastic cylinder that is freely rolling on a flat surface. The elastic properties of both the cylinder and the surface are $\nu = 0.3$ and $E = 200$ GPa. When the roller is at the location shown, the non-zero stress components at a depth $0.5a$, where a represents half of the length of contact as shown in the schematic, are given in the plot as the solid curves. Please answer the following questions:

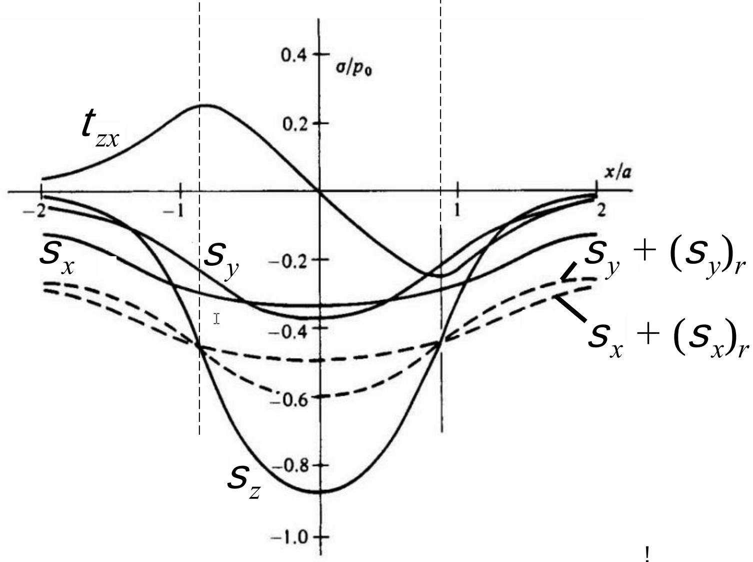
(a) What are the states of stress at locations A and B? Show your answers on stress elements with correctly labeled coordinate systems. Just read the values off the plot, noting that the stress axis is normalized by the peak pressure of the pressure distribution. Please keep stress in terms of these normalized values.



(b) Do the stress states at these two locations represent plane stress or plane strain? Explain and verify if you can.

(c) What are the principal stresses at these two locations?

(d) Which of these two locations is closer to reaching the yield point?



(e) If the yield point is reached at one or both of these locations, and several passes of the roller are made, then compressive residual stresses in the x- and y-directions, labeled $(s_x)_r$ and $(s_y)_r$, respectively, are generated. These altered stress components are shown as the dashed curves in the stress plot. How has the maximum shear stress at location B changed from the initial condition? Is it higher, lower, or unchanged? Justify your answer.

Problem #4

A curved beam is in the form of a semicircle as shown. One end is fully fixed and the other end is pinned to a block that can freely slide along a horizontal track. If a force F is applied on the block, find the maximum normal stress in the beam. The beam has a circular cross-section with a diameter d. State your assumptions, what you consider, and what you ignore.