PLEASE NOTE: Answer 3 out of the 4 problems. In case you answer the 4 problems, clearly state which 3 problems you want to be graded.

Problem #1

Two steel columns and one aluminum column are welded to two rigid plates, as shown in Figure 1. The three columns have the same initial length of 0.5 m and the same cross-sectional area of 40 mm^2. The steel is purely elastic with an elastic modulus of 200 GPa, and the aluminum is elastic-perfectly plastic with an elastic modulus of 70 GPa and a yield strength of 140 MPa. The structure is initially stress-free. The bottom plate is held fixed, and the top plate is subjected to a compressive load of F resulting in a vertical displacement of u. For each of the following cases, determine the function of $F(u)$ and the final stress states in the columns.

(a) F is increased to 20 kN, and then decreased to zero;

(b) F is increased to 50 kN, and then decreased to zero;

(c) F is increased to 100 kN, and then decreased to zero.

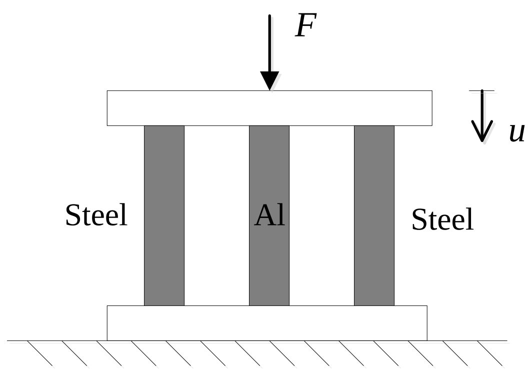
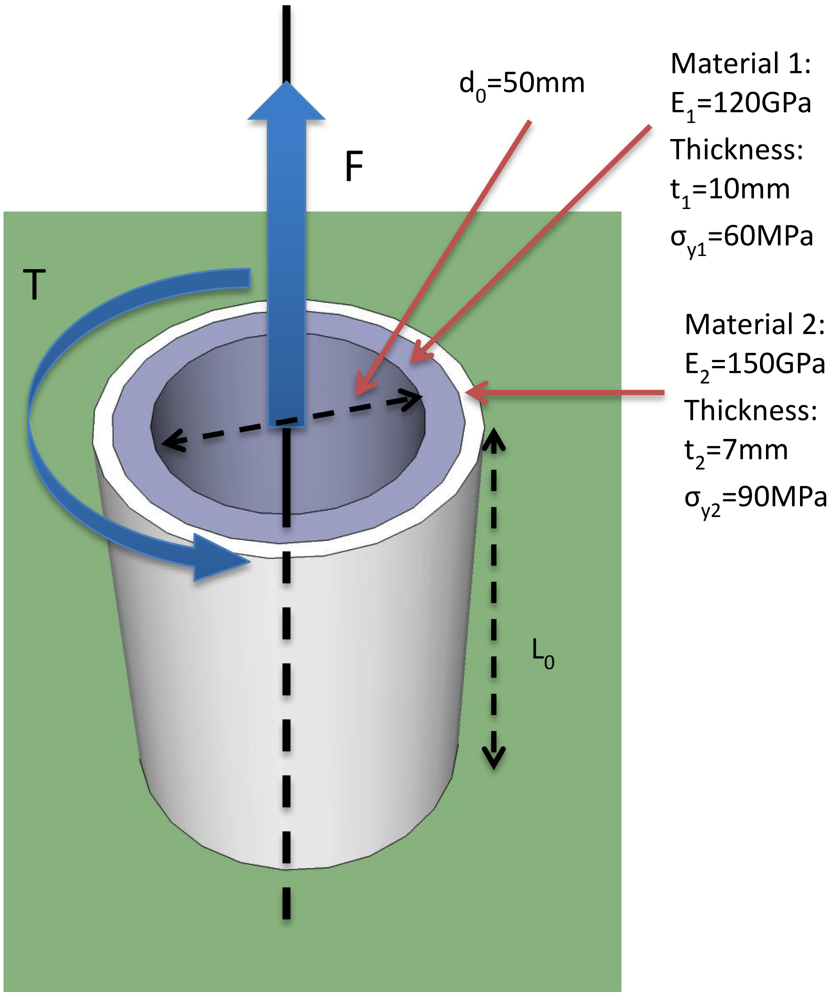


Figure 1

Problem #2

Consider the following thin walled tube simultaneously subjected to a tensile force F and a torque T. The two materials have an elastic perfect plastic behavior. All necessary information is given in the picture below:



1. Consider a first case in which the applied torque T is null. L is the instantaneous length of the tube. Construct, from a mechanical reasoning, a plot showing the evolutions of the applied Force with L (i.e. F as a function of L). Indicate and mathematically quantify the all key points on the graphics (i.e slopes, points where the slope changes).
2. Consider now a second case where T and F are non null. Show in a matrix form the components of stress which are non null. Express the relationship between the stresses in each material and the applied torque and force.
3. Using Tresca's yield criterion for both materials, construct a diagram for each material relating the maximum allowable torque to the maximum allowable force (i.e. T,F diagram). Indicate the expression of the points of intersections with the X and Y axis.
4. Where is the system more likely to yield plastically?

Problem #3

In a drop-weight test, a 1 in. diameter spherical ball weighting 0.15 lb is dropped freely on 0.120 in. thick $[0/\pm45/90]_s$ carbon fiber-epoxy beam specimens and the rebound heights are recorded. The beam specimens are simply supported, 0.5 in. wide and 6 in. long between the supports. The modulus of the laminate is 20 x 10^6 psi. The drop heights, rebound heights, and specimen deflections in three experiments are as follows:

Drop Height (ft)	Rebound Height (ft)	Measured Maximum Deflection (in)
1	0.72	0.056
4	2.2	0.138
6	3.02	0.15

a) Calculate the energy lost by the ball in each case
b) Calculate the maximum deflection of the beam in each case. Clearly list your assumptions
c) Compare the measured maximum deflection with what you calculated in b). Is there a difference, yes? No? Explain why.
d) How your answer in c) will change if the laminate is replaced with an isotropic material of the same modulus?

Problem #4

A new test method for fatigue testing of wires has been developed. It consists of a wire specimen that is bent 180° and is attached in a chuck connected to a motor on one end and in a bushing that freely rotates on the other end. As a result, the wire rotates along its long axis.

(a) Given the angular speed of the motor in terms of revolutions per minute (rpm), n, the wire diameter, d, the minimum radius of the bent wire, R_{min}, shown in the schematic, and the time to failure t_f (in minutes) for the test, please derive formulas for stress amplitude, σ_a, and cycles to failure, N_f, so a data point on the stress-life curve can be plotted for each wire tested. Assume the wire material is isotropic with elastic properties, E and ν.

(b) Can this approach be used to evaluate mean stress effects in the fatigue of wires? Please explain.