Ph.D. Qualifying Examination (Written)
Heat Transfer
Spring 2016

1. Consider a solid spherical metal ball of radius R, initially at a uniform temperature of T_i, which is suddenly immersed in a liquid at temperature T_l. The thermal conductivity (k), density (ρ), and specific heat (C) are known and constant. Assume the convection coefficient to be h, and the temperature dependence to be only of the form T(r,t):

(i) (20%) Write down the governing equations and necessary conditions for finding $\theta(r,t) = T(r,t) - T_l$.

(ii) (50%) Assuming $U(r,t) = r\,\theta(r,t)$, determine T(r,t). You do not need to evaluate any constants arising in the solution process.

(iii) (30%) Determine an expression for the ratio of the internal energy of the sphere at a time t, as a fraction of its initial internal energy.

The heat conduction equation in the spherical coordinates is given as:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial \emptyset}\left(\frac{\partial T}{\partial \emptyset}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

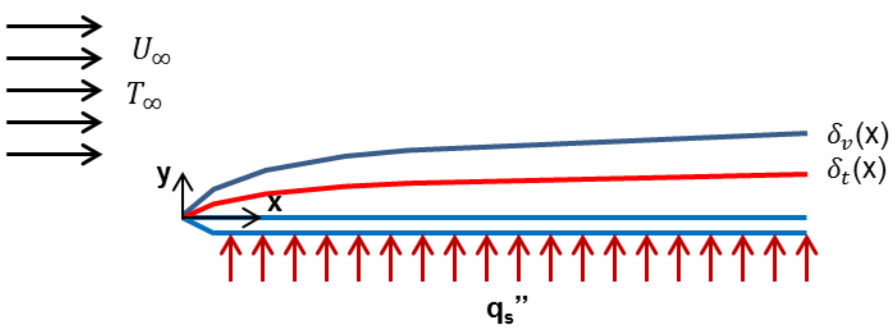
2. Consider the laminar forced convection boundary layer over a flat plate of length L, dissipating a uniform surface heat flux of q_s''. The horizontal and vertical components of the fluid velocity within the boundary layer are u(x,y) and v(x,y) respectively, fluid temperature T(x,y), and the surface temperature $T_w(x)$. The application of similarity theory reveals that:

$\eta = y\sqrt{\rho U_\infty/\mu x}$, $u(x,y)/U_\infty = f'(\eta)$, $f(\eta) = \psi(x,y)/\sqrt{\mu x U_\infty/\rho}$

$(T_w - T_\infty) = Nx^n$

$\phi(\eta) = (T(x,y) - T_\infty)/(T_w(x) - T_\infty)$

Here N and n are constants, ψ(x,y) the stream function, and other symbols have their usual meanings.



a. (10%) For the case illustrated in the figure is the Prandtl number 1, larger than 1, or smaller than 1 ?

b. (30%) Develop an expression for the thickness of the hydrodynamic boundary layer δ_v (x).

c. (40%) Find the value of n, and develop an expression for the average heat transfer coefficient for the plate of length L.

d. (20%) At x=L, find the normal distance y from the wall where the fluid velocity is 50% of the free stream velocity.

3. A molten droplet of a material with diameter D= 0.01 m travels along the centerline of a long tube at a constant velocity of V = 3 m/s. The droplet is at its melting temperature of 1700 K. It is diffuse, but not gray with a spectral absorptivity shown below. The air and the walls of the tube are at a temperature of 900 K.

1) Determine the emissivity of the droplet and list assumptions made.
2) Find the heat flux dissipated from the droplet during phase change as it falls through the tube assuming the conditions in 1) are met. The Nusselt number for convection over a sphere is given by:

$$Nu = \left(2 + \left(0.4Re_d^{1/2} + 0.06Re_d^{2/3}\right)Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{0.25}\right)$$

3) Determine an expression for how long it would take for the droplet to solidify, assuming a latent heat of fusion of h_{fg}=300 J/g and density of 8000 kg/m^3. If the tube is 10 m long, will the droplet completely solidify? (Surface Area of sphere = $4\pi r^2$; Volume of sphere = $4\pi r^3 / 3$)