Fall 2014, Ph.D. Qualifying Examination

Heat Transfer

1. The computer processor performance is highly sensitive to defects in the Silicon crystal lattice and therefore the devices must be constructed from single crystalline substrates. The substrates are commonly grown using the Czochralski process, which involves starting with a much smaller single crystal as a seed. The seed is inserted into an extremely finely temperature controlled vessel filled with liquid silicon, and is slowly pulled to remove heat from the liquid-solid interface, allowing more material to freeze, thus building up a larger single crystal (see following figure).

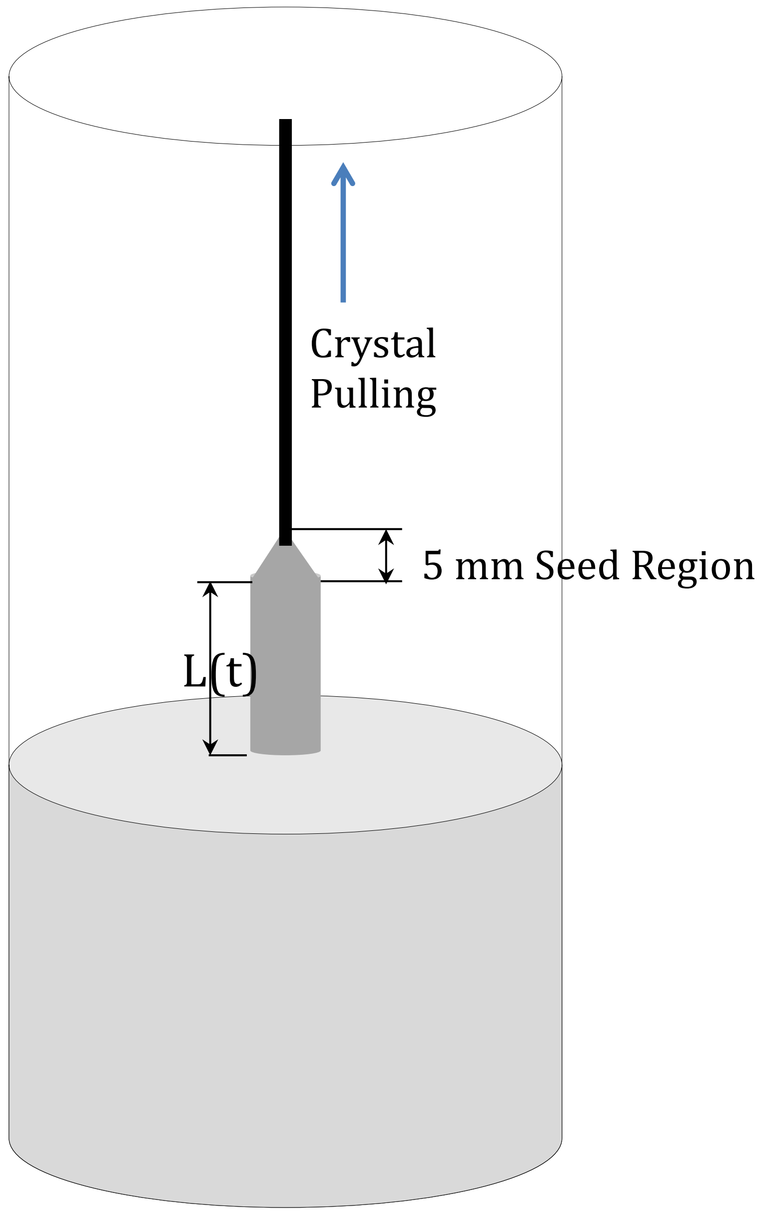
From experience, it has been determined that to form single crystal reliably, there must be a very small temperature gradient in the liquid near the solid/liquid interface, a maximum of 6 deg C/meter. Analyze the steady state condition, where the crystal is being continuously pulled and has a fixed cross-sectional area. The seed crystal is gradually cooled at a fixed rate to draw heat from the interface, which is at the melting temperature of silicon (T_m= 1414 deg C) and the interface remains stationary as the seed crystal is cooled and pulled upwards away from the interface. Assume there is negligible thermal contact resistance at the solid liquid interface.

[Part A – 4 points] Develop an expression for the length of the solidified crystal as a function of time L(t), see figure below, where the latent heat of fusion for silicon is 1926 kJ kg^{-1}, the density is 2570 kg m^{-3}, and the thermal conductivity of liquid silicon is 58 W m^{-1} K^{-1}. For this problem you can assume that there is negligible convection and radiation heat transfer, as the crystal is pulled out of the melt and cooled.

[Part B – 1 point] Determine the time it takes to pull a 20 cm long cylindrical single crystal.

[Part C – 3 points] Develop an expression for the seed temperature as a function of time at steady state, assume the initial crystal length is 5 mm, which is when the cross-sectional area has reached its final value (see figure below).

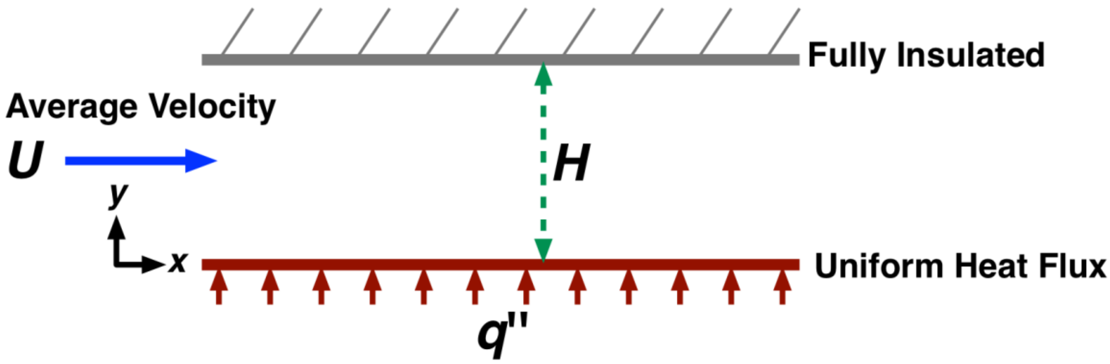
[Part D - 2 points] If the thermal conductivity of solid silicon near the melting point is 30 W/mK, determine the seed temperature at the end of the 20 cm cylindrical crystal growth, neglecting any radial temperature variations.



2. Consider the case of laminar fluid flow with average velocity *U* between two infinite parallel plates separated by height *H*. The fluid is heated through the bottom plate with a uniform heat flux *q''*, and the top plate is completely insulated. The flow is thermally and hydraulically fully developed and steady, axial conduction and viscous dissipation are negligible, and fluid properties are constant.

The fully developed velocity profile for this flow is given by:

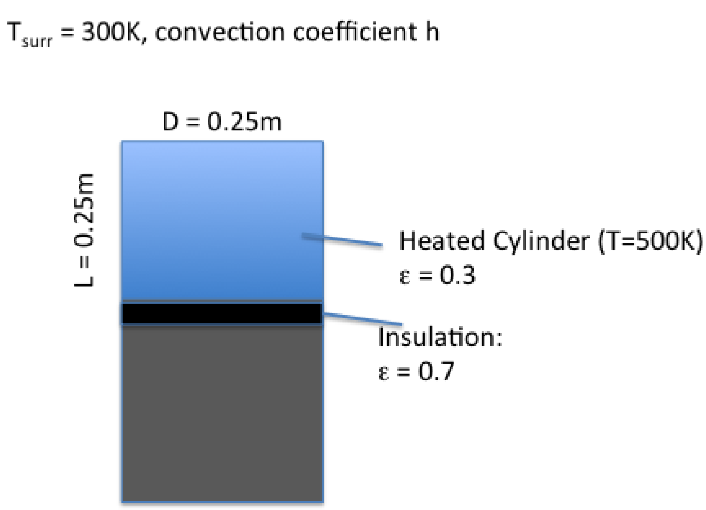
$$u(y) = 6\bar{U}\left[\frac{y}{H} - \frac{y^2}{H^2}\right]$$



a) Sketch representative fully developed velocity and temperature flow profiles for this flow.
b) Derive the fully developed temperature profile for this flow.
c) Derive the fully developed Nusselt number (Nu = $h D_h / k$) for this flow.

3. A thin wall cylinder (open at both ends) of diameter D=0.25m and height L=0.25m is placed on an insulated post with the top open to the atmosphere. The surfaces of the cylinder and insulation are consider diffuse, opaque, and gray. The emissivity of the cylinder material is ε=0.3 and the insulated bottom is ε =0.7. Power is applied to the cylinder in order to maintain the surface temperature at 500K while the surrounding room is at 300K. Neglecting convective heat losses, find the amount of power applied to the cylinder to maintain this temperature.

If convection with a coefficient of h is allowed to dissipate heat from the surfaces, develop the equations needed to solve for the amount of power required for the cylinder. Do not solve but give the necessary equations in symbolic notation.



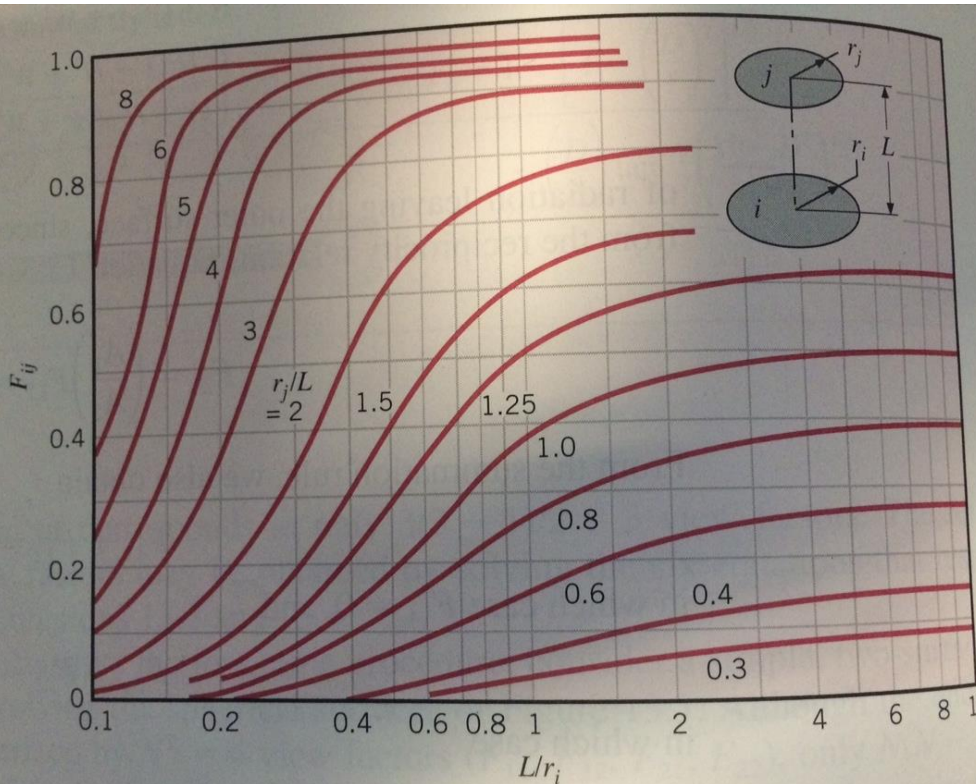


FIGURE 13.5 View factor for coaxial parallel disks.