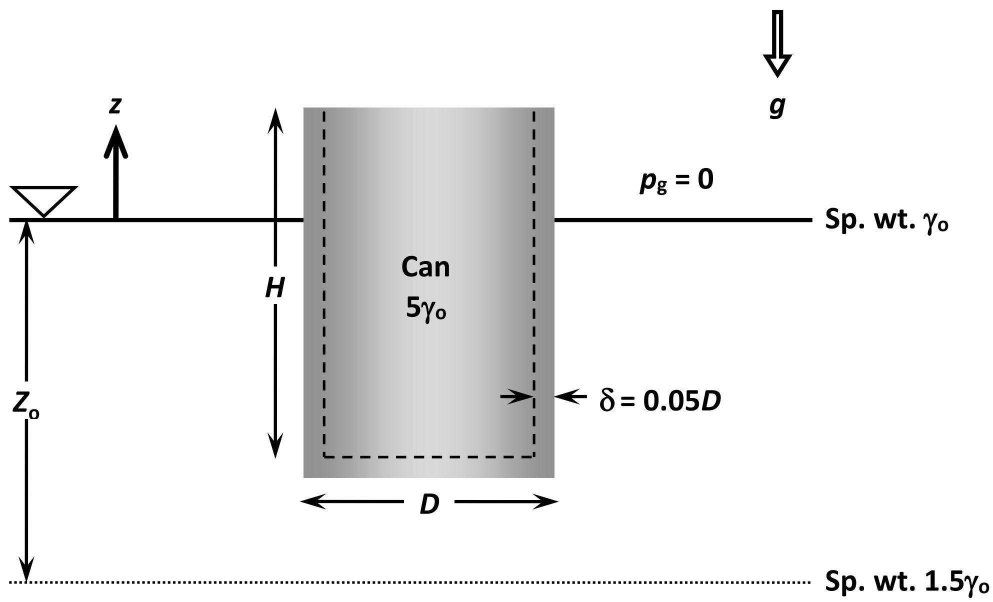
1. A large open tank is completely filled with a linearly stratified liquid whose density depends linearly with depth z so that the specific weight of the liquid is γ_o at the free surface and $1.5\gamma_o$ at a depth Z_o.

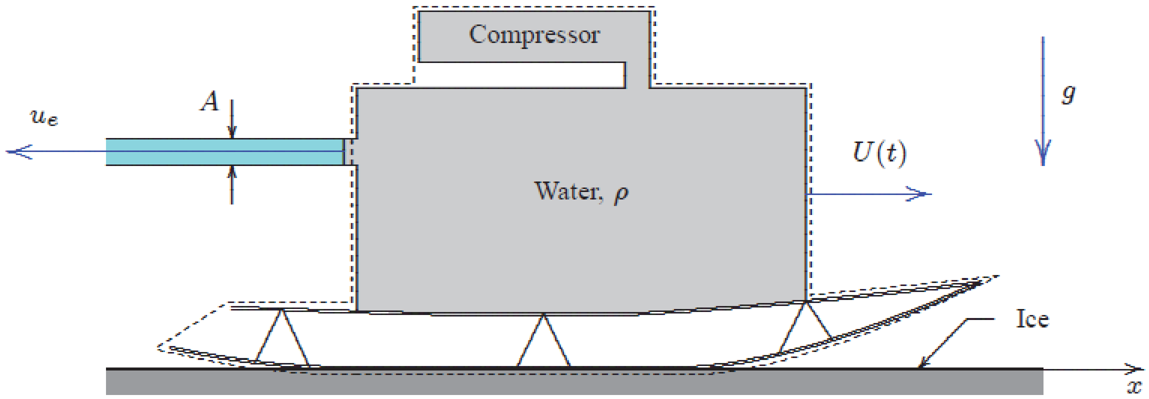


a) What is the gage pressure distribution in the liquid assuming that the liquid is at rest and the gage pressure at the free surface $p_g = 0$?

b) A cylindrical container (a "can") open at the top and made of a material of specific weight $5\gamma_o$ with an outer diameter D, inner height H and wall thickness $\delta = 0.05D$ is gently and slowly placed in the tank. What is the volume of liquid that overflows from the tank if the can floats in the tank and is stable in this configuration?

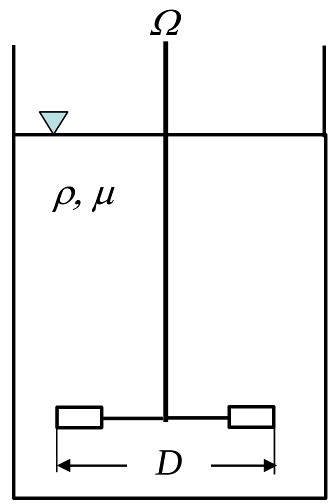
c) What is the minimum mass M that must be (slowly and gently) added to the can to ensure that the can will be completely submerged?

2. High pressure in a tank mounted on a sled is maintained by a compressor so that the water of density ρ leaving the tank through the orifice does so at a constant speed, u_e, relative to the sled. The orifice area is A. The instantaneous mass of the sled, water, tank and compressor is $M(t)$. The magnitude of the friction force between sled and ice is μMg, where μ is the coefficient of kinetic friction and g is gravitational acceleration. The sled starts from rest with an initial mass, M_o.



Ignoring aerodynamic forces on the sled, develop an expression for the instantaneous speed, $U(t)$, in terms of u_e, M, M_o, μ, g and time, t.

3. An immersed spinning impeller (having an angular velocity Ω and diameter D as shown in the sketch below) is used to stir liquid of density ρ and viscosity μ from rest to nominally-steady motion. In addition to these parameters, the power P that is imparted by the impeller also depends on the characteristic time τ that is needed for the stirring (which can be measured relative to the beginning of the process). Before a full scale prototype is constructed, it is necessary to assess the performance of the stirrer by building a 1/10 scale model using a liquid of viscosity μ_m and density ρ_m (it is given that $\nu_p = 0.1\nu_m$, and $\rho_p = 0.9\rho_m$).



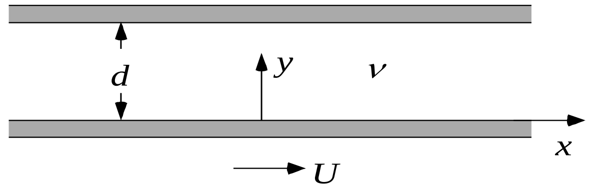
a) Derive a set of dimensionless groups to determine the dependence of P on the other variables. Use ρ, τ, and μ as repeating (primary) parameters.

b) Using your dimensional analysis, determine the power imparted by the impeller in the prototype stirrer P_p if the power in the model stirrer is P_m.

c) Is it possible to estimate the ratio between the characteristic times for stirring in the prototype and in the model? Which stirs faster? ***Explain briefly***.

d) It is also possible to use ρ, Ω, and D as repeating variables to derive a (different) set of dimensionless groups. How would the relationship between P_p and P_m change? ***Explain briefly***.

4. A pair of infinitely long, parallel plates separated by a distance d have a viscous fluid of kinematic viscosity ν in the space between them. The plates and fluid are initially at rest. At time $t=0$, the lower plate impulsively starts moving to the right with constant speed U. The two-dimensional Navier-Stokes and continuity equations, in Cartesian coordinates without body force, are given by



$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

a) *Simplify* the equations appropriate to this situation, *scale* them using appropriate scales for length and velocity and d^2/ν for the time scale, and *pose the correct boundary and initial conditions*, in dimensionless form.

b) *Solve* the problem of part **a)** by the method of *separation of variables*. You need not compute the integrals necessary to determine the final coefficients.

c) In the absence of an upper plate (with the fluid now extending to infinity in the y-direction), this problem is identical to Stokes' 1st Problem (also known as the "Rayleigh plate"), which admits a similarity solution using the dimensionless similarity variable $\eta = y/\sqrt{\nu t}$. Determine by substituting this variable into your simplified dimensional equations and supplementary conditions whether the problem of part **a)** also admits a similarity solution.