1. A Newtonian incompressible fluid with viscosity μ and density ρ fills the gap between two coaxial cylinders. The outer cylinder rotates with a constant angular velocity Ω, whereas the inner cylinder is stationary. The radii of inner and outer cylinders are kR and R, respectively, where $k < 1$. Determine the velocity distribution for a laminar flow. Find the torque required to turn the outer cylinder. Analyze your solutions for the velocity and torque in the limiting case of $k \to 0$. End effects and gravity effects can be neglected.

Continuity equation in polar coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (r\rho v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

Linear momentum equations in polar coordinates:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \rho g_r + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \rho g_z + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right)$$

Shear stress in polar coordinates:

$$\tau_{r\theta} = \mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right), \quad \tau_{\theta z} = \mu\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right), \quad \tau_{rz} = \mu\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)$$

2. Consider a cylindrical buoy of shell thickness T that is submerged symmetrically between two immiscible liquids "A" and "B" (having densities ρ_A and ρ_B) as shown in Figure 1 below. It can be assumed that the interface between the liquids "A" and "B" remain horizontal and stable (i.e., heavy and light liquids do not move down or up, respectively). The buoy shell contains air at standard conditions (it may be assumed that $\rho_{air} << \rho_A, \rho_B$), and the buoy is guided by a thin wire passing through the shell such that there is no friction between the buoy and the wire and no leaks.

a) It is desired to consider ***two*** density configurations namely, $\rho_A > \rho_B$, and $\rho_A < \rho_B$. Determine what should be the weight of the buoy W_b for ***each*** configuration so that its elevation is unchanged.

b) The buoy is gently pushed up until it is completely submerged in liquid "A" and then it is released. For the buoy weights computed in part (a) determine whether the buoy moves up, down, or remains stationary when $\rho_A > \rho_B$, ***and*** when $\rho_A < \rho_B$. Please explain your answers briefly based on the physical considerations.

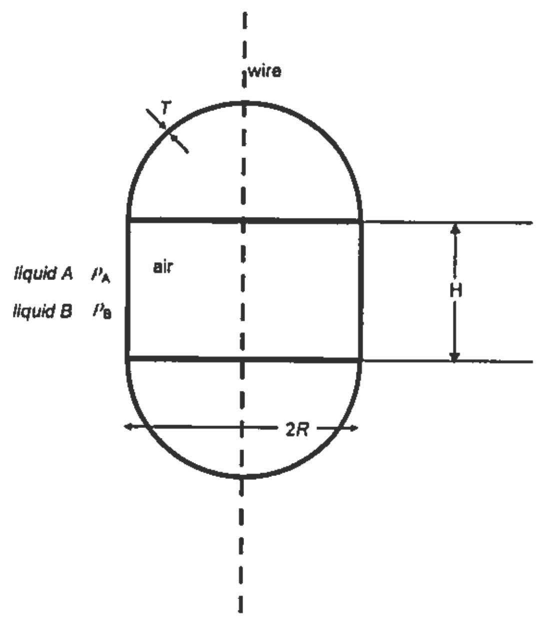
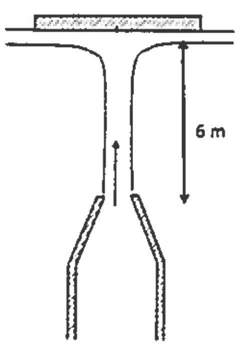


Figure 1



3. A vertical jet of water leaves a nozzle at a speed of 15 m/s. The diameter at the nozzle exit is 25 mm. The jet suspends a circular plate of diameter 85 mm at a constant height of 6 m above the nozzle exit. What is the mass of the plate?

4. The one-dimensional transport of mass in a chemical reactor can be described by the following diffusion equation:

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = \mathcal{D}\frac{\partial^2 C}{\partial x^2} - kC$$

where C is the dimensionless concentration of a chemical species, t is time, u is the velocity component along the x direction, $\mathcal{D}$ is the diffusion coefficient, and k is the reaction rate. Traditionally, $\partial C / \partial t$ is known as the accumulation term, $u(\partial C / \partial x)$ is known as the advection term, $\mathcal{D}(\partial^2 C / \partial x^2)$ is known as the diffusion term, and $-kC$ is known as the decay term.

a) What are the units (in the $MLT\Theta$ basic dimension/primary quantities system) of the terms in the diffusion equation, the diffusion coefficient $\mathcal{D}$, and the reaction rate k?

b) Nondimensionalize the terms in this equation using a characteristic length scale of L and a characteristic velocity scale of V. Please give a brief physical description of the dimensionless groups, or Π terms, in this nondimensionalized diffusion equation.

c) If the (spatial) dimensions of the chemical reactor are doubled, but the velocity scale remains unchanged, how will the relative importance of the various terms in this nondimensionalized diffusion equation change?