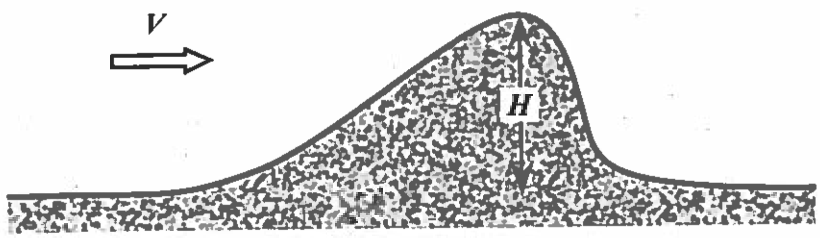
1. Consider the formation of a sand dune on a flat desert plain due to wind.



a) Use dimensional analysis to determine how the height of the dune H depends upon the average wind speed V, the length of time that the wind has been blowing τ, the average weight W and diameter d of the sand grains, and the density ρ and kinematic viscosity ν of air.

b) If the interface between the sand and the air on the surface of the dune is "fully" rough, the height of the dunes becomes independent of ν. Measurements for a sand dune with a fully rough surface show that H is proportional to W^{-1}. Determine how the height of a dune with a fully rough surface then depends upon the other relevant parameters.

c) Your measurements tell you that $H_0 = 0.2$ m for a given type of sand when the wind blows for $\tau_0 = 24$ h. Can you determine how the height of a dune of the same type of sand with a fully rough surface will change if the wind blows instead for $\tau_1 = 48$ h? If so, determine the height of the dune H_1.

2. Consider a cylindrical can of weight W and an internal cross section A that is floating vertically in liquid of density ρ such that its sealed (top) end is a distance h above the free surface and its open end is facing downward as shown in Figure 2a below. The can is supported by trapped air and a free cylindrical piston separates between the liquid and the trapped air. It can be assumed that the weight and thickness of the piston are negligible, and that its motion along the inner surface of the container is frictionless. It can also be assumed that the trapped air follows Boyles law, $p \cdot V =$ const where p and V are the pressure and volume.

a) Determine in the force F that is necessary to hold the can as shown in Figure 2b.

b) Does the holding force F change if the can is pushed deeper into the liquid pool after it is fully submerged in Figure 2b? If so, explain how and why.

c) Does the holding force F in part (b) change after the can is fully submerged if the can has an internal seal that prevents additional motion of the piston as shown in Figure 2c? If so, explain how and why.

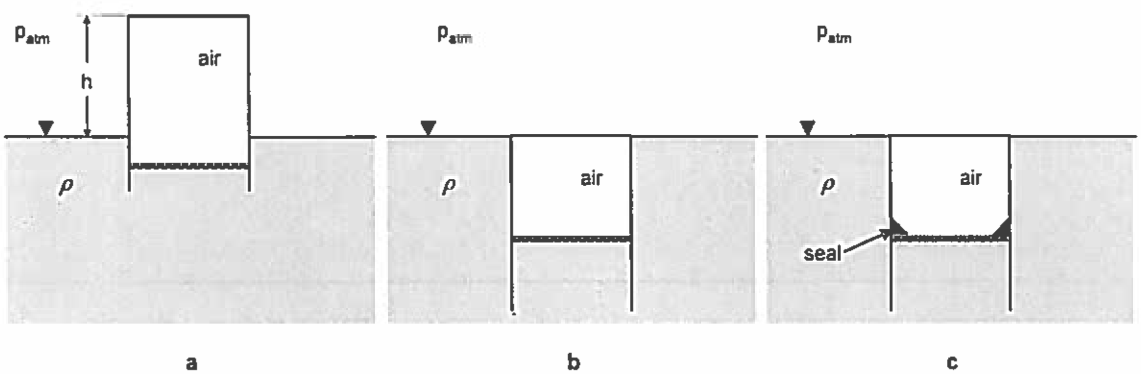
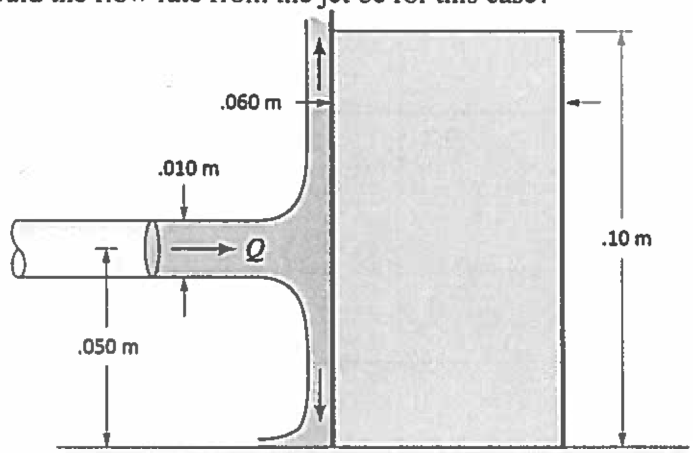


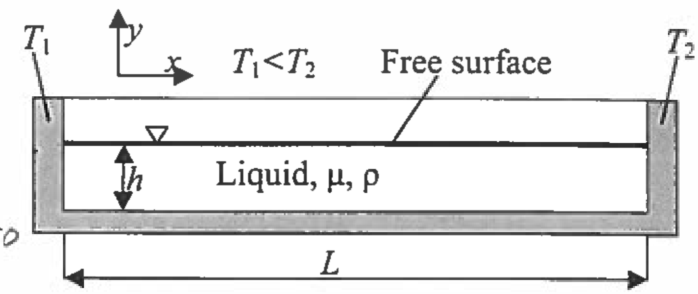
Figure 2

3. A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (60 mm by 200 mm by 100 mm) that weighs 6 N as shown below. The jet can deliver a maximum flow rate of 10 gal/min. (1 gal = 3.785 $*10^{-3}$ m^3)

a) Determine the minimum height above the ground that the jet must hit the block to tip it, assuming the block cannot slide. What should the flow rate from the jet be for this case?
b) Determine the minimum height above the ground that the jet must hit the block to tip it before it slides, assuming a static coefficient of friction between the block and the ground of $\mu_s = 0.7$ (i.e., the force necessary to overcome static friction is $R_x = 6$ N x μ_s). What should the flow rate from the jet be for this case?



4. Since surface tension of liquids is normally decreases with temperature, when a liquid is exposed to a temperature gradient, a surface tension gradient emerges along the liquid free surface. The surface tension gradient results in a net interfacial force that generates liquid motion, commonly referred TO as a Marangoni flow.



Consider a shallow cavity with length L, opposite walls of which are maintained at different temperatures T_1 and T_2, as shown in the figure. The cavity is filled with a viscous, incompressible, Newtonian fluid with depth h. Assume that the temperature changes linearly along the cavity and that surface tension is a linear function of temperature, i.e. $\sigma = \sigma_0 + \sigma_T(T - T_0)$, therefore at the free surface $\mu \frac{\partial u}{\partial y} = \frac{\partial \sigma}{\partial x}$. Here, σ_0 is surface tension at temperature T_0, and $\sigma_T < 0$ is the surface tension coefficient. Gravity and the end effects at the corners of the cavity can be neglected, and density does not change with temperature.

a. Formulate the boundary conditions at the free surface and the bottom wall,
b. Find the steady state velocity distributions in the liquid,
c. Find the liquid pressure gradient in the liquid.

Continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

Linear momentum equations

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$