GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2020

_____________DESIGN_____________
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

✸ Please sign your **name** on the back of this page —

ID Number________________________

QUESTION I. – DESIGN METHODOLOGY (10 points)

DESIGN PROBLEM – PUMPKIN CHUCKIN' DEVICE

Your team has decided to try to win the Vermont Pumpkin Chuckin' Festival and set a Guinness Book of World Record as a class project. You are required to follow the guidelines of a systematic design methodology and turn in a report documenting the main deliverables as follows.

Key Rules (source: http://vtpumpkinchuckin.blogspot.com/p/rules.html)

- Max. total weight of trebuchet is 500 lbs.
- Max. height is 10 feet
- Projectile = 5 pound pumpkin minimum
- Device must be safe. The following can be added for safety:
    - A cocking mechanism may be used without counting towards the total weight.
    - May be mounted or assembled on a trailer.
    - No digging into the ground is allowed (like a pit for the counterweight to fall further). You may use the ground to stake the trebuchet to prevent rocking. Ropes, wires or straps are also allowed and are not included in the total weight.
    - Winches or other electrical motors are allowed to cock a trebuchet.

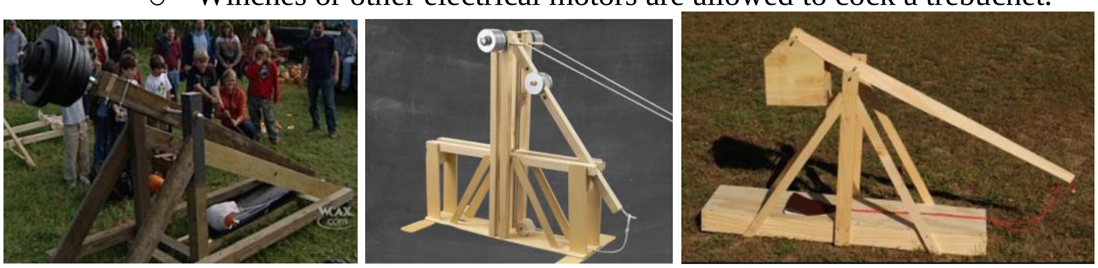

Figure 1: Example Pumkin Lauching Devices

DELIVERABLES (YOU ARE REQUIRED TO ELABORATE THESE ISSUES)

1.1 *Design Process:*

(a) Name two design methods that are likely to be particularly effective for this particular problem and explain why these methods would be highly effective for your team's design and goals. **(2 pt.)**

(b) Outline a complete design process, listing the different methods needed, for this problem from understanding the problem to the initial alpha prototypes or proof of concept models. Explain why each method is particularly suited to this problem in one or two sentences. Include at least 5 design methods. You do not need to generate solutions **(6 pt.)**

1.2 *Conceptual Design:*

(a) Create a functional diagram(s) that characterizes the overall function and its decomposition into sub-functions. You may pick a particular design architecture, if needed for the functional method. You are required to use one of the formal function analysis tools (examples include, but are not limited to EMS models, Functional Basis, FBS, IDEF0, FAST, UML). List the name of the method. **(2 pt.)**

ID Number________________________

Question II A. (10 points)

Design Embodiment for an Improved Air Conditioner

Air conditioners are appliances that make modern life comfortable through air temperature and humidity control, but home air conditioners can be frustrating to maintain. Failure can cause water damage, gas leakage, electrical fire hazards, and discomfort. Let's try to improve their design to offer the best-in-class reliability and performance, with less concern for cost.

Within the air conditioner, a motor is connected via an output shaft to a fan to blow air throughout the house, as shown:

1. **Describe, qualitatively, the time dependent loading on the on the motor output shaft during operation. (1 point)**

2. **What failure modes can occur under this type of loading? (1 point)**

3. **Please estimate the location of the peak stress on the motor output shaft. (1 point)**

4. **Describe some design features of output shafts of fan motors in poorly designed air conditioners. (2 points)**

5. **In addition to shaft failure, the motor bearings in the fan can fail. What type of bearings would you choose for your motor to have accuracy and long-life, and why? (2 points)**

6. **Please diagrammatically illustrate the type of bearings indicated in the answer to Question 5, their location, orientation. Justify your decisions. (2 points)**

7. **What shortcuts do you anticipate that companies that make poorly designed air conditioners have made in their bearing design to reduce cost? (1 point)**

Question II B. Component Design Analysis (10 points)

Water Pressure Regulator

In Figure 1, a Watts 1 LF25AUBZ3 water pressure regulator is shown installed in a residential water line in a residential home. A water pressure regulator (sometimes called a *pressure-reducing valve*, or PRV) is a dome-shaped brass fitting that generally is found just past the main shutoff valve, where the main water line enters the house. This valve brings down the pressure to a safe level before the water reaches any plumbing fixtures inside the home. Most home plumbing fixtures are designed to work best at a pressure of about 50 psi (345 kPa), but it is not uncommon for municipal water supplies to enter the home with pressures as high as 150 or 200 psi (1035-1379 kPa). A water pressure regulator is essential in situations where the municipal water supply enters the home at a very high pressure, or where water pressure is irregular.

Figure 1 - Watts LF25AUB-Z3 water pressure regulator installed in residential water pipe (water flows from left to right)

A cross-sectional schematic drawing of the internal components of the Watts 1 LF25AUBZ3 water pressure regulator is shown in Figure 2. Water pressure regulators usually have an adjustment screw on top (item 53 in Fig. 2). Inside, a water pressure regulator has a variable spring-loaded diaphragm (item 20 in Fig. 2) that automatically widens and narrows depending on the amount of water pressure entering the valve. When the water enters the regulator at high pressure, the inner mechanism constricts the diaphragm to narrow the flow of water. This can reduce the pressure into a range of 50 to 80 psi (345-552 kPa), greatly reducing the stress on pipes and fixtures installed past the valve. Conversely, when the incoming water pressure drops, the diagram opens wider to allow more water to flow through the valve. The adjustment screw on the top of the regulator can be tightened to increase the tension on the inner spring (item 52 in Fig. 2), thereby reducing the pressure of the water as it exits the valve, or loosened to allow water to flow more freely through the valve (thereby increasing the outgoing water pressure). The Watts Series LF25AUB-Z3 water pressure reducing valve shown in Figures 1 and 2 is suitable for water supply pressures up to 300psi (2,068 kPa) and may be adjusted from 25 – 75psi (172 – 517 kPa).

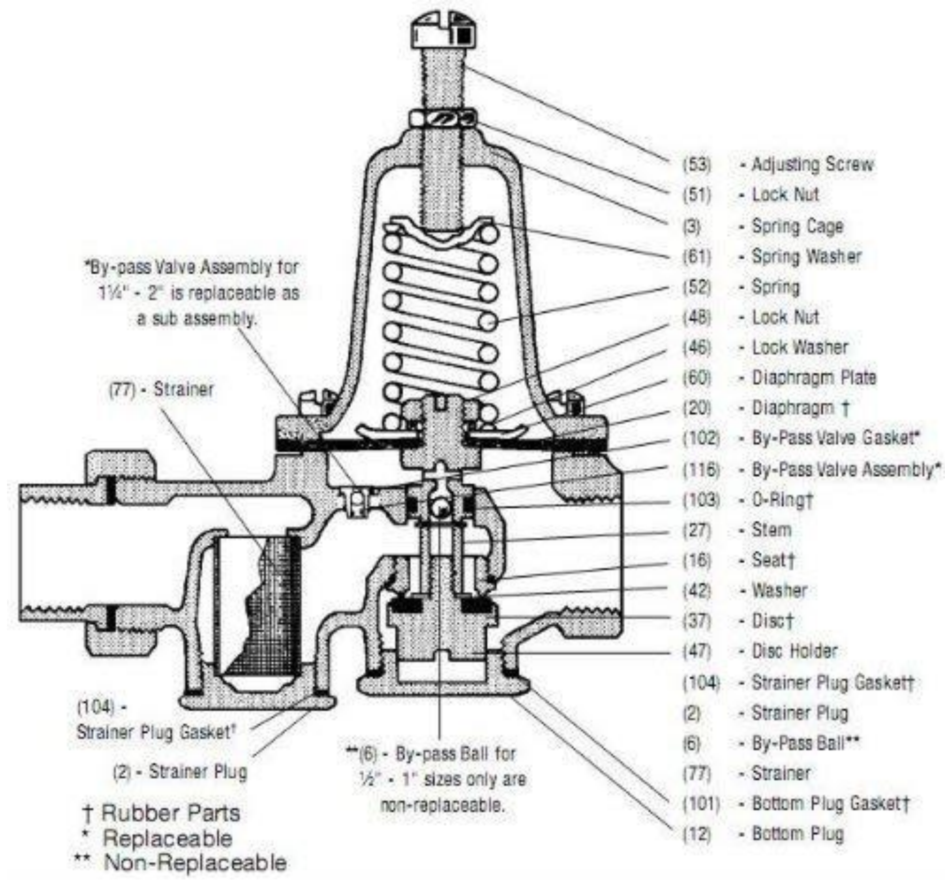

Figure 2 - Watts LF25AUB-Z3 water pressure regulator cross-sectional schematic view

The spring shown in Figure 2 (item 52) is a critical component of a water pressure regulator. This particular helical compression spring has 12 total coils with squared and ground ends. The wire diameter of the spring is 4 mm and the mean coil diameter is 25 mm. The spring's free length Lf = 150 mm. The spring is made of A228 music wire with a modulus of rigidity G=79.3 GPa and modulus of elasticity E = 207 GPa.

a) If we assume that the maximum allowable water pressure is 2,100 kPa, what is the maximum force on the spring? (2 points)

b) Assuming the maximum force on the spring is 3,150 N, calculate the factor of safety against completely closing the spring to its solid length? (4 points)

As shown in Figures 1, 2 and 3, the pressure regulator contains a top cover (labeled "3 spring cage" in Figure 2) that houses the spring. This cover is fastened to the main body of the pressure regulator by six fully threaded slotted head machine screws. Each screw is a steel M6 class 4.6 screw with a tensile stress area of 20.1 mm^2 and proof strength of 585 N/mm^2.

The thickness of the diaphragm is 2 mm and the thicknesses of the brass members in the bolted joint are 5 mm each. The modulus of elasticity E for carbon steel is 207 GPa. The modulus of elasticity E for brass is 97 GPa.

For the following questions, assume that the total load on the joint is 3,150 N.

c) Assuming a bolt constant of 0.2, calculate the minimum pre-load F_i required for each screw to avoid joint separation (2 points)

d) Assuming a preload of 1000 N per screw, calculate the factor of safety against static failure of the screws. Will the joint fail? (2 points)

Design Qualifying Exam Equation Sheet - Page 1

$$n_d = \frac{\text{loss-of-function strength}}{\text{allowable stress}} = \frac{S}{\sigma(\text{or } \tau)}$$

$$V = \frac{dM}{dx}$$

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + \left(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2\right)\sigma - \left(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2\right) = 0$$

$$\frac{dV}{dx} = \frac{d^2M}{dx^2} = q \qquad \tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\int_{V_A}^{V_B} dV = V_B - V_A = \int_{x_A}^{x_B} q\,dx \qquad \sigma = E\epsilon \qquad \epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\nu\frac{\sigma_x}{E}$$

$$\int_{M_A}^{M_B} dM = M_B - M_A = \int_{x_A}^{x_B} V\,dx$$

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$\tau_{yx} = \tau_{xy} \qquad \tau_{zy} = \tau_{yz} \qquad \tau_{xz} = \tau_{zx}$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi \qquad \sigma = \frac{F}{A} \qquad \sigma_x = -\frac{My}{I} \qquad I = \int y^2 dA$$

$$\tan 2\phi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \sigma_{\max} = \frac{Mc}{I} \qquad \sigma_{\max} = \frac{M}{Z}$$

$$\frac{\sigma_x - \sigma_y}{2}\sin 2\phi_p - \tau_{xy}\cos 2\phi_p = 0 \qquad \tau = \frac{VQ}{Ib} \qquad \tau = G\gamma \qquad \tau = \frac{V}{A}$$

$$\tan 2\phi_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \qquad \tau = \frac{T\rho}{J} \qquad \tau_{\max} = \frac{Tr}{J} \qquad \theta = \frac{Tl}{GJ}$$

$$\frac{\sigma_x - \sigma_y}{2}\cos 2\phi_p + \tau_{xy}\sin 2\phi_p = 0 \qquad E = 2G(1+\nu) \qquad \epsilon_x = \epsilon_y = \epsilon_z = \alpha(\Delta T)$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \qquad \sigma = -\epsilon E = -\alpha(\Delta T)E$$

$$H = T\omega \qquad T = 9.55\frac{H}{n}$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad H = \frac{FV}{33\,000} = \frac{2\pi Tn}{33\,000(12)} = \frac{Tn}{63\,025}$$

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases}$$

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

$$P_{cr} = \frac{\pi^2 EI}{l^2}$$

$$k_e = 1 - 0.08\, z_a$$

$$\sigma_{max} = K_f \sigma_0 \quad \text{or} \quad \tau_{max} = K_{fs}\tau_0$$

$$k_a = a S_{ut}^b$$

$$K_t = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$

$$P_{cr} = \frac{C\pi^2 EI}{l^2}$$

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

$$\frac{P_{cr}}{A} = S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2 \frac{1}{CE} \qquad \frac{l}{k} \le \left(\frac{l}{k}\right)_1$$

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{shear}(K_{ts} - 1)$$

$$\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \ge S_y$$

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q_{shear} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2}$$

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} \qquad \sigma' \ge S_y$$

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2}$$

$$\sigma' = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

$$K_{fm} = K_f \qquad K_f|\sigma_{max,o}| < S_y$$

$$K_{fm} = \frac{S_y - K_f\sigma_{ao}}{|\sigma_{mo}|} \qquad K_f|\sigma_{max,o}| > S_y$$

$$K_{fm} = 0 \qquad K_f|\sigma_{max,o} - \sigma_{min,o}| > 2S_y$$

$$S_f = a\, N^b \qquad a = \frac{(f S_{ut})^2}{S_e}$$

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b} \qquad b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$$

$$S_f \ge S_{ut} N^{(\log f)/3} \qquad 1 \le N \le 10^3$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

$$\sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right| \qquad \frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

mod-Goodman $\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}} = \dfrac{1}{n}$

Langer static yield $\sigma_a + \sigma_m = \dfrac{S_y}{n}$

DE-Goodman

$$\frac{1}{n}=\frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2+3(K_{fs}T_a)^2\right]^{1/2}+\frac{1}{S_{ut}}\left[4(K_f M_m)^2+3(K_{fs}T_m)^2\right]^{1/2}\right\}$$

$$\sigma'_{\max}=\left[(\sigma_m+\sigma_a)^2+3(\tau_m+\tau_a)^2\right]^{1/2}$$

$$=\left[\left(\frac{32K_f(M_m+M_a)}{\pi d^3}\right)^2+3\left(\frac{16K_{fs}(T_m+T_a)}{\pi d^3}\right)^2\right]^{1/2}$$

$$\omega_1=\left(\frac{\pi}{l}\right)^2\sqrt{\frac{EI}{m}}=\left(\frac{\pi}{l}\right)^2\sqrt{\frac{gEI}{A\gamma}}$$

$$D_{\max}=D+\Delta D \qquad D_{\min}=D$$

$$d_{\max}=d+\delta_F \qquad d_{\min}=d+\delta_F-\Delta d$$

$$d_{\min}=d+\delta_F \qquad d_{\max}=d+\delta_F+\Delta d$$

$$d=\left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2+3(K_{fs}T_a)^2\right]^{1/2}+\frac{1}{S_{ut}}\left[4(K_f M_m)^2+3(K_{fs}T_m)^2\right]^{1/2}\right\}\right)^{1/3}$$

$$F_b=P_b+F_i=CP+F_i \qquad F_m<0$$

$$F_m=P_m-F_i=(1-C)P-F_i \qquad F_m<0$$

$$P_m=P_b\frac{k_m}{k_b} \qquad T=KF_i d$$

$$n_p=\frac{S_pA_t}{CP+F_i} \qquad n_L=\frac{S_pA_t-F_i}{CP}$$

$$n_0=\frac{F_i}{P(1-C)}$$

$$F_p=A_tS_p \qquad \sigma_a=\frac{C(P_{\max}-P_{\min})}{2A_t}$$

$$n_f=\frac{S_a}{\sigma_a} \qquad n_f=\frac{S_e(S_{ut}-\sigma_i)}{S_{ut}\sigma_a+S_e(\sigma_m-\sigma_i)}$$

$$n_p=\frac{S_p}{\sigma_m+\sigma_a}$$

$$\sigma_m=\frac{(F_{b\max}+F_{b\min})/2}{A_t}=\frac{(CP_{\max}+F_i)+(CP_{\min}+F_i)}{2A_t}$$

$$\sigma_m=\frac{C(P_{\max}+P_{\min})}{2A_t}+\frac{F_i}{A_t}$$

$$F_i=\begin{cases}0.75F_p & \text{for nonpermanent connections, reused fasteners}\\ 0.90F_p & \text{for permanent connections}\end{cases}$$

$$n_y=\frac{S_y}{\sigma'_{\max}} \qquad \tau=K_B\frac{8FD}{\pi d^3}$$

$$k\doteq\frac{d^4G}{8D^3N} \qquad y_{cr}=L_0C'_1\left[1-\left(1-\frac{C'_2}{\lambda^2_{\text{eff}}}\right)^{1/2}\right]$$

$$\lambda_{\text{eff}}=\frac{\alpha L_0}{D} \qquad L_0<\frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

$$C'_1=\frac{E}{2(E-G)} \qquad L_0<2.63\frac{D}{\alpha}$$

$$C'_2=\frac{2\pi^2(E-G)}{2G+E}$$

$$L_0=2(D-d)+(N_b+1)d=(2C-1+N_b)d$$

$$S_{ut}=\frac{A}{d^m} \qquad \sigma_A=F\left[(K)_A\frac{16D}{\pi d^3}+\frac{4}{\pi d^2}\right]$$

$$(K)_A=\frac{4C_1^2-C_1-1}{4C_1(C_1-1)} \qquad C_1=\frac{2r_1}{d}$$

$$(K)_B=\frac{4C_2-1}{4C_2-4} \qquad C_2=\frac{2r_2}{d}$$

$$N_a=N_b+\frac{G}{E} \qquad \tau_B=(K)_B\frac{8FD}{\pi d^3}$$

$$F=F_i+ky$$

$$FL^{1/a}=\text{constant} \qquad F_1L_1^{1/a}=F_2L_2^{1/a}$$

$$C_{10}=F_R=F_D\left(\frac{L_D}{L_R}\right)^{1/a}=F_D\left(\frac{\mathcal{L}_Dn_D60}{\mathcal{L}_Rn_R60}\right)^{1/a}$$

$$\frac{F_e}{VF_r}=1 \quad \text{when } \frac{F_a}{VF_r}\le e$$

$$\frac{F_e}{VF_r}=X+Y\frac{F_a}{VF_r} \quad \text{when } \frac{F_a}{VF_r}>e$$

$$n_3=\left|\frac{N_2}{N_3}n_2\right|=\left|\frac{d_2}{d_3}n_2\right| \qquad e=\frac{n_L-n_A}{n_F-n_A}$$

$$H=T\omega=(W_td/2)\,\omega \qquad V=\pi dn/12$$

$$W_t=33\,000\frac{H}{V} \qquad W_t=\frac{60\,000H}{\pi dn}$$

Tables/Figures to be provided on the exam if needed by Chapter in Shigley's Mechanical Engineering Design 9th Edition

Chapter 4
Tables: 4-2

Chapter 6:
Tables: 6-2, 6-3, 6-4, 6-5,
Figures: 6-20, 6-21

Chapter 7
Tables: 7-6, 7-9,

Chapter 8
Tables 8-1, 8-2, 8-9, 8-10, 8-11, 8-17

Chapter 10
Tables: 10-1, 10-2, 10-4, 10-5, 10-6, 10-7

Chapter 11
Tables: 11-1, 11-2, 11-3

Appendix tables: A-11, A12, A13, A-14, A-15-1 through A-15-16, A-18, A-20