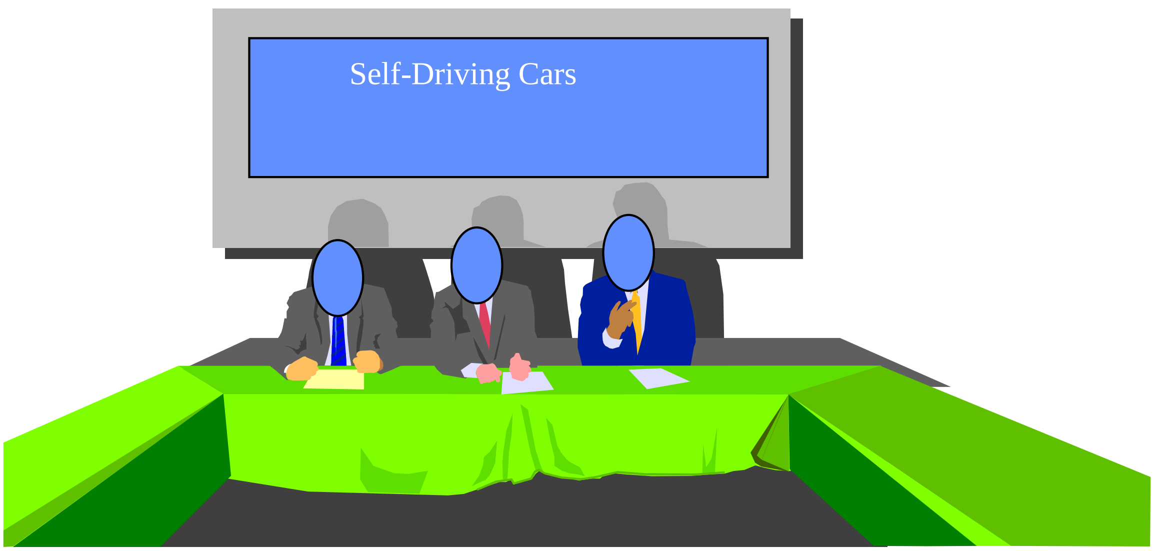
COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – SPRING 2016

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

S.-K. Choi, C. Paredis, D. Rosen, S. Sitaraman (Chair), and Y. Wang



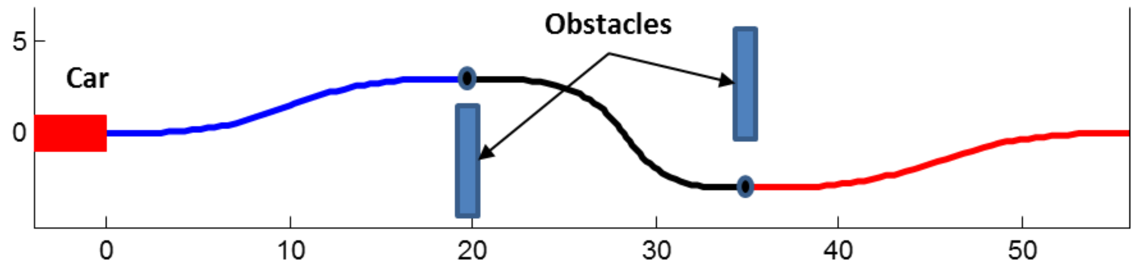
- All questions in this exam have a common theme: ***Self-Driving Cars***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

Self-driving cars need to sense their environments and update their planned trajectories quickly so that other cars and obstacles can be avoided, while ensuring a smooth, comfortable ride. This problem will address issues of trajectory planning by investigating curve models of driving paths.

A driving situation is illustrated below, with a car on the left and two obstacles. A proposed path is provided. As a first approximation, a 3-segment composite curve is proposed. To ensure a smooth path, C^2 continuity is assumed among curve segments, since instantaneous changes in rate of steering cannot be accomplished. The curve endpoints are illustrated using the large dots along the curve. Units are in meters.



Answer the following questions:

a) Given that C^2 continuity is desired between curve segments, what are appropriate geometric models for the curves. Justify your recommendations.

b) Sketch the control polygons for the three curve segments illustrating how you achieved C^2 continuity (revise your recommended models in problem a) if necessary).

c) Assuming that a composite Bezier curve is chosen to model the path, derive the equation for one segment of the curve (i.e., expand the equation for $p(u)$ for your chosen n). You do not need to substitute values for control vertices or simplify the equation.

$$\boldsymbol{p}(u) = \sum_{i=0}^{n} \mathbf{p}_i B_{i,n}(u)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Let's use a simpler model to approximate the paths and require only C^1 continuity. The control vertices for two cubic Bezier curves with C^1 continuity are $\mathbf{p}_0$ = (0, 0), $\mathbf{p}_1$ = (5, 0), $\mathbf{p}_2$ = (10, 5), $\mathbf{p}_3$ = (15, 3), $\mathbf{q}_0$ = (15, 3), $\mathbf{q}_1$ = (?, ?), $\mathbf{q}_2$ = (20, -4), $\mathbf{q}_3$ = (30, -2).

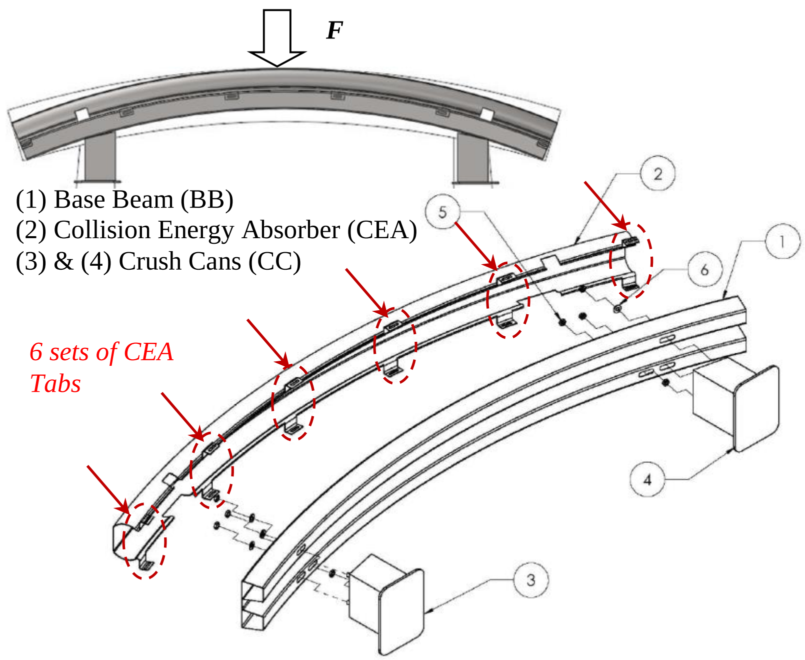
d) Compute the coordinates of $\mathbf{q}_1$ to achieve C^1 continuity.

e) Sketch the control polygons and the curves, given these control vertices.

f) Compute the point on the first curve ($\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$) at u = 0.5.

Question 2 – Finite-Element Analysis

As of July 2015, 14 self-driving cars have been involved in minor traffic accidents on public roads. Eight of these accidents involved being rear-ended at a stop sign or traffic light.



To optimize the bumper system design shown in the figure, FEA is applied for crashworthiness test. The main structures include a *Base Beam*, a *Collision Energy Absorber* (CEA), and two *Crush Cans*. The *Base Beam* and CEA are connected by 6 sets of *CEA Tabs*. The external load is simplified as a force (***F***) applied at the middel point of the CEA.

(a) Assume the *Base Beam* is rigid, build a finite-element model to estimate the deformation where the load ***F*** is applied. Make necessary assumptions of lengths, cross-section areas, materials, etc.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.

(b) Discuss briefly how the model would be different if the *Base Beam* is not rigid.

<u>Element A - Stiffness Matrix</u>

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l=(x_2-x_1)/L$ and $m=(y_2-y_1)/L$ are directional *cos*() and *sin*() respectively.

<u>Element B - Stiffness Matrix</u>

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

where E, I, and L are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Analysis

To improve the performance of electric vehicles and self-driving cars, car manufacturers have introduced the newest magnetic technologies of the time. For instance, the traditional damper has been replaced with a cylinder filled with a magnetic liquid surrounded by an electric coil. This idea originated from the concept for an electromagnetic suspension first patented in 1934.

In general, a magnetic suspension system is governed by Equations (1) and (2). Equation (1) represents the electric behavior:

$$Ri(t) + L\frac{di(t)}{dt} = V_c \tag{1}$$

where V_c, R, I, and L are the control voltage, the resistance of the coil, the coil current, and the coil inductance, respectively. Equation (2) represents the mechanical behavior:

$$m\frac{d^2y(t)}{dt^2} = -\frac{i^2(t)}{y(t)} + mg \tag{2}$$

where y, m, and g denote the separation distance, the mass of the suspended object, and the gravitational constant.

(**a**) Write down a state-space representation (first-order equations) for $i(t)$ and $y(t)$ to describe the magnetic suspension system.

(**b**) The identified state-space representation from (a) includes a nonlinear term. Linearize the state-space representation using a first-order Taylor series expansion around an operation point with $y(\mathrm{t}) = y_0$.

(**c**) What are the potential issues with the linearization process for obtaining numerical solution of nonlinear differential equations? What are the benefits of considering the linearization in the process?

(**d**) By using the obtained state-space representation from (b), determine approximate values of the solutions at the time t=0.2 s. Apply Heun's method with a step size of h=0.2. Assume that g=10 m/s^2, m=0.2 kg, L=5 Henry, V_c = 20 V and R=10 Ω. Also, at t=0 s, y_0 = 0.1 m, y_0'= 0.05 m/s, and i_0 = 0.1 amp.

(**e**) When an extremely small value is used for the step size, how do you expect the solution to change? What are the potential issues you can think of? How can you avoid these issues without changing the step size already used? Provide a detailed explanation.

The equations for Heun's method are:

$$\begin{cases} y_{i+1} = y_i + h\phi \\ \phi = (k_1 + k_2)/2 \\ k_1 = f(t_i, y_i) \\ k_2 = f(t_i + h, y_i + hk_1) \end{cases}$$