COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2013

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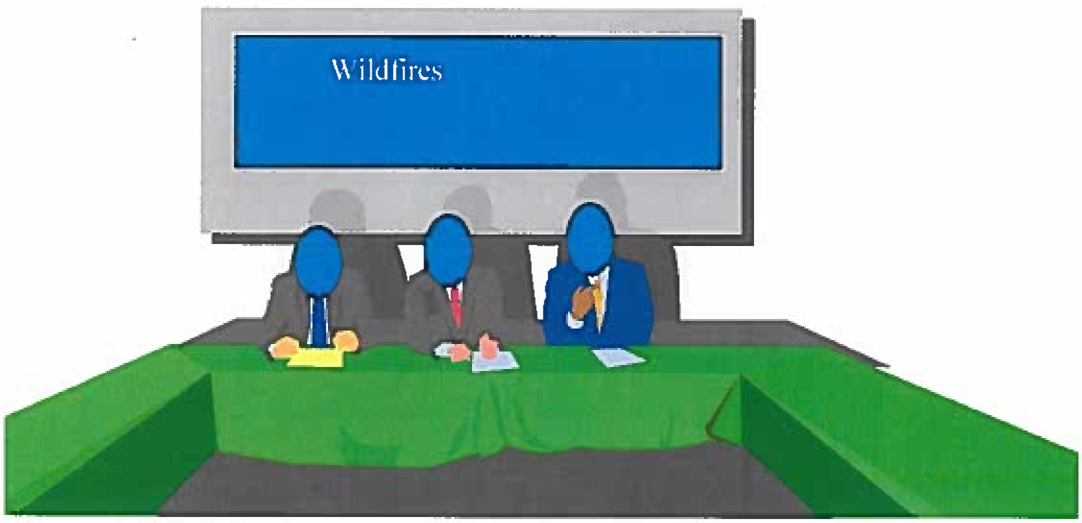


- All questions in this exam have a common theme: ***Wildfires***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

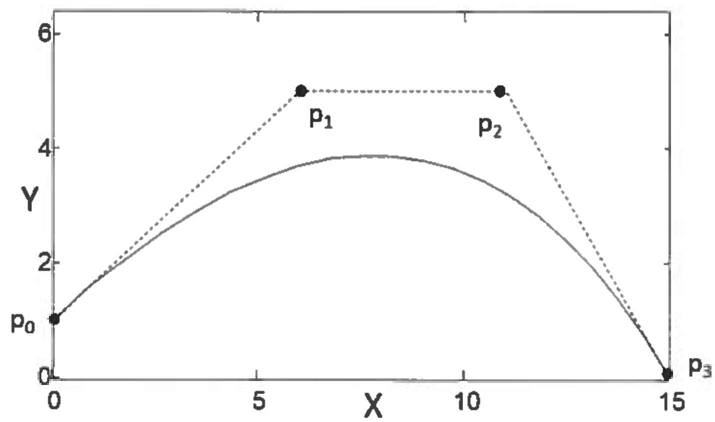
Question 1 - Geometric Modeling

In this problem we will model the path taken by water that is being sprayed by a fire hose that is being held by a firefighter. Assume that the sprayed water follows the path given in the figure below, which can be approximated by the Bezier curve indicated by the given control vertices.

Control vertices are as follows: $\mathbf{p}_0 = (0, 1, 0)$, $\mathbf{p}_1 = (4, 6, 0)$, $\mathbf{p}_2 = (9, 5, 0)$, $\mathbf{p}_3 = (15, 0, 0)$, assuming that the units are in meters. Answer the following questions:

a) Derive the equation for the cubic Bezier curve that passes through the given control vertices. Simplify the equations into the form: $\mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0 = \mathbf{k}(u)$

b) Compute the point on the curve at $u = 0.5$.



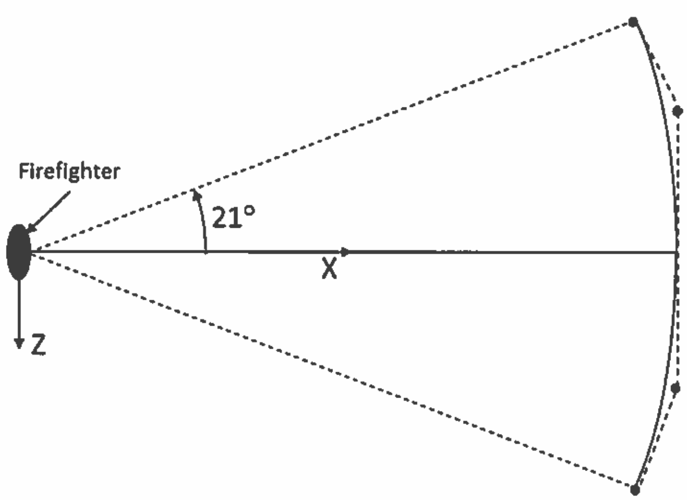
Now, assume that the firefighter wants to spray water along a curve, not just at point $\mathbf{p}_3$. The firefighter rotates back-and-forth by 21 degrees in each direction (42 degrees total). You are to derive the equation for the Bezier surface patch that is traced by the water. See the figure at bottom (top view).

c) What degree is the Bezier surface patch?

d) Explain how you can compute the coordinates of all the control vertices on the surface patch. Write down the equation(s) that enable you to compute the CV coordinates. You need not carry out all of the multiplications.

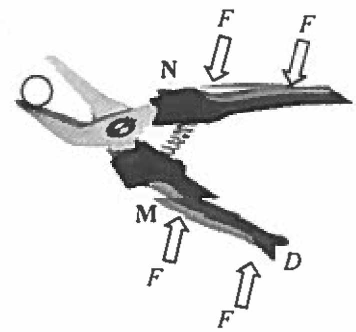
e) Derive the equation of the surface patch. You can use symbols for the CVs and you do not need to multiply out the equation or simplify it.

f) At what angle does the water strike the ground?



Question 2 – Finite-Element Analysis

As a design engineer of a wildfire fighting equipment manufacturer, you are assigned a task to design a shear rescue tool, as shown in the figure on the right.



a) Based on the given external hand-hold forces, build a simplified finite-element model for one grip, grip **M**, for its structural analysis.
b) From your model, formulate an assembled stiffness matrix for grip **M** to estimate the deformation at the end point *D*. Make necessary assumptions of lengths, cross-section areas, etc.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.

Element 1 - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l=(x_2-x_1)/L$ and $m=(y_2-y_1)/L$ are directional *cos()* and *sin()* respectively.

Element 2 - Stiffness Matrix

$$[K]=\frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

where *E*, *I*, and *L* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Methods

The behavior of forest fires can be characterized in terms of their rate of propagation, which depends on sudden changes in direction due to the available combustible material and the effect of weather on the fire, etc. In established forest fires, the proportion of the total width of the forest which has been destroyed is denoted by L. The rate of change with respect to time t (in hours) is called the destruction rate. Investigations show that the destruction rate in x and y directions can be modeled by the following equations:

$$\frac{dL_x}{dt} = 998\,L_x - 1998\,L_y \quad \text{..............................(1)}$$

$$\frac{dL_y}{dt} = 1000\,L_x - 2000\,L_y \quad \text{..............................(2)}$$

a) Assume that at t=0, L_x=1.0 and L_y=2.0. Determine approximate values of the solution at the time t=0.2 hour by using Euler's method with a step size of h=0.1 hour.

b) Now, apply Heun's method for solving the given system equation with the same step size of h=0.1 hour.

c) Comment on the accuracy of your answers from (a) and (b) by comparing with the exact solution of $L_x(t) = -\frac{1998}{998}e^{-2t} + \frac{2996}{998}e^{-1000}$ and $L_y(t) = -\frac{1000}{998}e^{-2t} + \frac{2996}{998}e^{-1000}$. In order to maintain the stability of a solution, suggest a maximum allowable step size for Euler's method.

d) As smaller and smaller step sizes are used, are you expecting to have good accuracy improvements in both Euler's and Heun's methods? Provide a detailed explanation.

Note: The Euler formula is $x_{n+1} = x_n + hf_n$

The equations for Heun's method are:

$$\begin{cases} y_{i+1} = y_i + h\phi \\ \phi = (k_1 + k_2)/2 \\ k_1 = f(t_i, y_i) \\ k_2 = f(t_i + h, y_i + hk_1) \end{cases}$$