COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – FALL 2012

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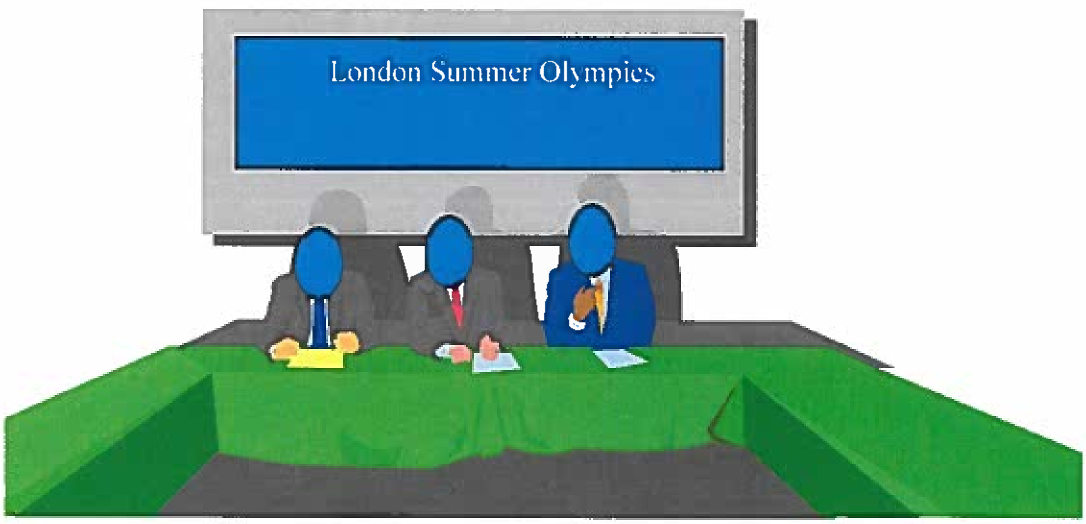


- All questions in this exam have a common theme: ***London Summer Olympics***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Problem 1 - Geometric Modeling

The path of a soccer ball will be modeled in this problem. Assume that a player was fouled and a direct free kick was awarded at the spot of the foul, directly in the center-line of the field and 40 meters from the opposing goal. Further, assume that the opposing players form a wall 10 meters from the spot of the foul. The kicker must kick the ball over the wall in order to score.

A schematic of the kick is shown below in the XZ plane. Note that the ball travels in 3 dimensions. We will model the path of the ball as a cubic Bezier curve. The height of control vertex $\mathbf{p}_1$ should be chosen so that the ball clears the opposing player wall; we will assume that if the ball has a height of 2 m at the wall, then the ball will pass over the wall.

Control vertices are as follows: $\mathbf{p}_0 = (0,0,0)$, $\mathbf{p}_1 = (12, -1, z_1)$, $\mathbf{p}_2 = (32, 0, 3)$, $\mathbf{p}_3 = (40, 2, 2)$.

Answer the following questions:

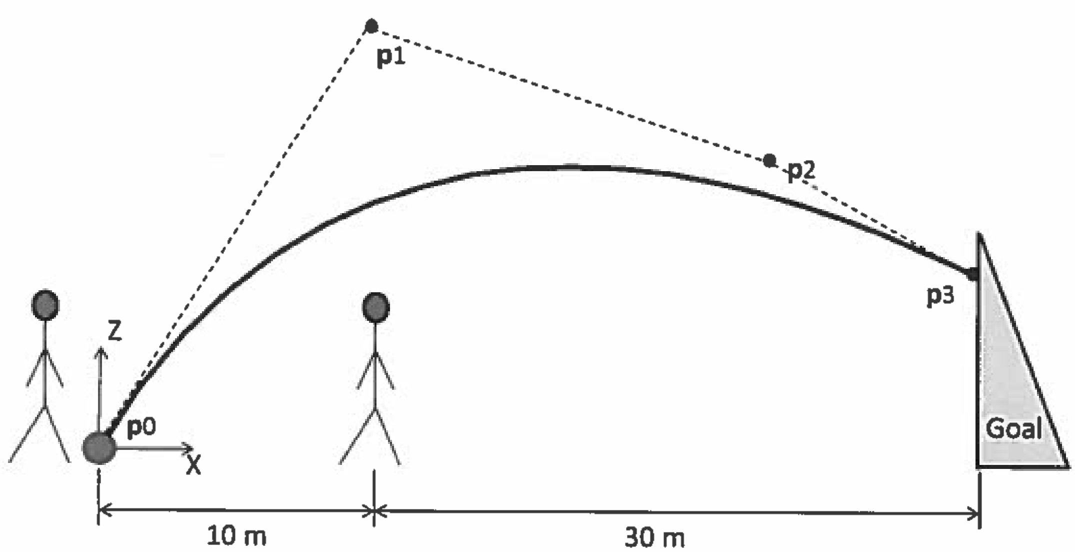
a) Derive the equation for the cubic Bezier curve that passes through the given control vertices. Simplify the equations into the form: $\mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0 = \mathbf{k}(u)$

b) If $z_1 = 3$, compute the point on the curve at $u = 0.3$.

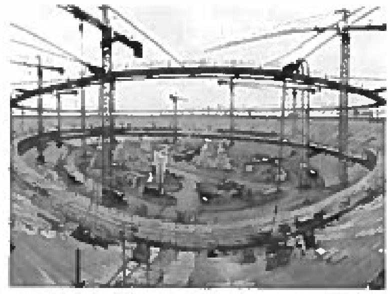
c) Given that the curve must pass through the point $x_1 = 12$, $z_1 = 2$, describe a procedure for finding the value of z_1. Provide a problem formulation and present the steps needed. Identify any numerical methods you would use.

d) Consider only the $z(u)$ equation that you derived in part a). If $u = 0.25$, what value of z_1 enables the ball to be 2 m high? That is, set $u = 0.25$ and $z(u) = 2$ m, then compute z_1.

e) Sketch the path of the ball in the XY and XZ planes. Include the control polygon in both graphs.

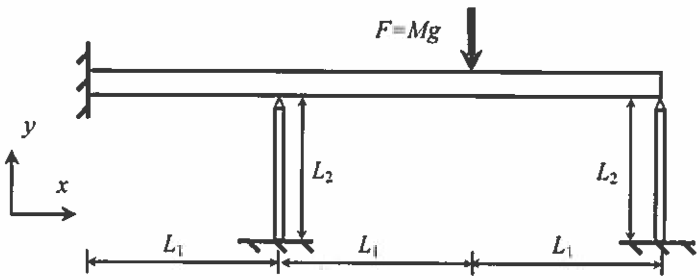


Problem 2 – Finite-Element Analysis



During the construction of the main Olympic stadium, many support structures were used, shown in the figure on the right. Your job as the design engineer is to double check the integrity.

In the simplified model shown below, the horizontal structure has a length of $3L_1$, a cross-section area of A_1, and a moment of inertia I_1. The two supporting vertical structures both have a length of L_2, a cross-section area of A_2, and a moment of inertia I_2. The supporting positions are at L_1 distance away from the fixed left-end and the right-end of the horizontal structure. Notice the assumed *point contacts* between the horizontal and vertical structures. Both horizontal and vertical structures are made of a material with a modulus of elasticity E. A load with a mass of M_1 is assumed to be applied vertically at a distance L_1, from the right end.



You are asked to analyze the structure using finite-element formulation.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.
5. Determine the vertical deflection of the horizontal structure at the loading position, assuming that the structure does not touch the point contact. Show all steps to find results. No need to calculate the final values.

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; C and S are directional *cos*() and *sin*() respectively.

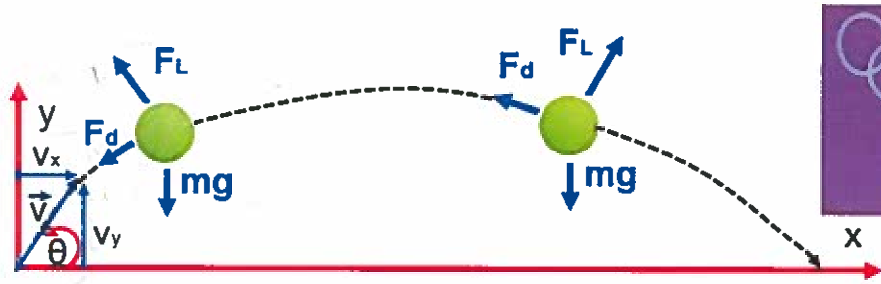
Element B - Stiffness Matrix

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

where E, I, and L are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Problem 3 – Numerical Methods

In the 2012 Olympic Games in London, Andy Murray won the Tennis gold after producing one of his best performances. One of the key components of controlling the tennis ball trajectory is spinning the ball. A ball with backspin tends to float through the air and falls to the tennis court at a shallow angle. Conversely, a ball with topspin falls at a steeper angle. The force due to spin is called the magnus force (F_L) and it always acts at right angles to the drag force (F_d) and to the spin axis. Suppose that a tennis ball of mass m and radius R is traveling at speed v and at an angle Θ upwards from the horizontal. The forces acting on the ball are the gravitational force mg downwards, the drag force $F_d = kC_d mv^2$ backwards and the magnus force $F_L = kC_L mv^2$ upwards and at right angles to the path of the ball (assuming the ball has backspin) as shown in the figure below. In this problem, the lift coefficient C_L=0.2, the drag coefficient C_d=0.5, and $k = 0.03$, and m=0.057 kg.



(a) Obtain the equation of the motion for the tennis ball trajectory. Write down two first-order differential equations in terms of the velocity in the horizontal direction (v_x) and the velocity in the vertical direction (v_y).

(b) Assuming that at t=0, the initial velocity v_0=20m/s, the launch angle $\Theta_0 = 30°$, and the initial height y_0=1.0 m. Determine approximate values of the solution at the point t=0.2 by using the Euler method with the step size of h=0.1.

(c) Now, ignore the spin effect (C_L=0) and solve for a purely vertical hit straight up (assuming that v_0=15m/s, y_0=1.0 m, and $\Theta_0 = 90°$). Obtain the approximate velocity at t=0.2 by applying the Runge-Kutta method (RK4) with h=0.2.

(d) How does a good Runge-Kutta method determine the step size such that a user-specified desired accuracy can be guaranteed?

Note: The equations for RK4 method are:

$$x_{n+1} = x_n + (h/6)(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4})$$

$$k_{n1} = f(t_n, x_n),\ k_{n2} = f(t_n + (h/2),\ x_n + (h/2)k_{n1}),\ k_{n3} = f(t_n + (h/2),\ x_n + (h/2)k_{n2})$$

$$k_{n4} = f(t_n + h,\ x_n + hk_{n3})$$

The Euler formula is $x_{n+1} = x_n + hf_n$