GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Spring Semester 2020

COMPUTER-AIDED ENGINEERING
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

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COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – SPRING 2020

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
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- All questions in this exam have a common theme: ***Prosthesis***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

As a product engineer, you are given a task to design a prosthetic leg that will be fabricated using a 3D printer. The prosthetic leg is shown in Figure 1.1. You are asked to use Bezier surface patches to model the prosthetic leg.

Figure 1.1: Prosthetic Leg

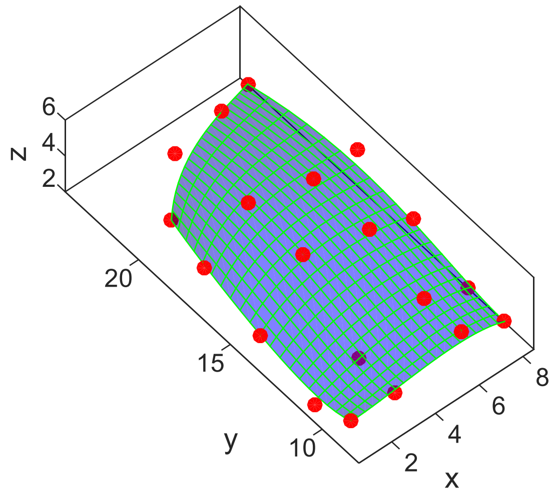

Figure 1.2: Control Points for Patch 1

(A) You received the design of Patch 1 (Figure 1.2) with the control points as:

Patch 1:

(2 20 3)	(3 21 5)	(6 22 4)	(8 23 3)
(1 17 4)	(3 17 6)	(6 17 5)	(8 17 5)
(1 14 3)	(3 14 6)	(6 14 5)	(8 14 4)
(1 11 2)	(3 11 3)	(6 11 4)	(8 11 3)
(1 9 3)	(3 9 3)	(6 9 4)	(8 9 3)

What is the normal direction vector at point (1, 9, 3)? Briefly explain why you can calculate in your way.

(B) To start the design of Patch 2, we have decided some of the control points as follows:

Patch 2:

(-5 22 -4)	(-3 22 -4)	**P_4**	(2 20 3)
(-5 17 -6)	(-3 17 -5)	**P_5**	**P_1**
(-5 14 -5)	(-3 14 -5)	**P_6**	**P_2**
(-5 11 -4)	(-3 11 -4)	**P_7**	**P_3**
(-5 9 -4)	(-3 9 -5)	**P_8**	(1 9 3)

To ensure the C^0 and C^1 continuity between the two patches, what are the coordinate values of **P_1**, **P_2**, **P_3**, **P_4**, **P_5**, **P_6**, **P_7**, and **P_8**? Briefly explain how you decide the values and why you can calculate in your way.

(C) Derive the generic equation for one cubic-quartic (3^{rd}-4^{th}order) Bézier surface patch (similar to Patch 1 or Patch 2) in the matrix form.

2) Finite-Element Analysis

A running prosthesis (Figure 2.1) can help those with below-knee amputees to gain active lifestyle. It consists of a foot adapter and a spring foot. The spring foot is critical to the performance of the runner. It has a curved shape and is typically made by carbon fiber composites to allow enough flexibility for the conversion between strain energy and kinetic energy. Understanding the mechanical deformation of the spring foot is a critical aspect of the design. You are asked to do a simple calculation of the deformation of the spring foot by using finite-element analysis.

The spring foot can be simplified into a structure as shown in Figure 2.2. The structure uses a material with a Young's modulus of E and a cross-section area of A. The goal is to find the displacement of point 1 where a load of W is applied.

Figure 2.1: Running Prosthesis

2.1 What kind of boundary conditions would you apply?

2.2 Write down the element stiffness matrices for the elements used in the model. Use minimum number of elements.

2.3 Assemble element stiffness matrices into a global stiffness matrix.

2.4 Modify the global matrix to introduce the boundary conditions.

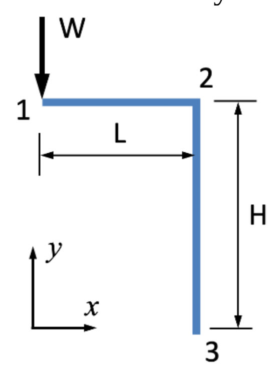

Figure 2.2: Design of the spring foot.

Element Stiffness Matrix

$$[K] = \begin{bmatrix} AE/L & 0 & 0 & -AE/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -AE/L & 0 & 0 & AE/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix}$$

where E is the Young's modulus, A is the cross-sectional area, L is the length, and I is the moment of inertia for the element.

3) Numerical Analysis

Running prostheses are typically comprised a C-shaped carbon fiber spring attached to a rigid socket on the residual leg of an individual with lower limb loss (see figure to the right). These type of running blade prostheses have allowed particularly athletic individuals with amputation to compete on nearly even grounds with Olympic athletes in sprinting. One of the key mechanical properties of these devices is the effective stiffness (expressed in kN/m) of the compound carbon fiber strut. The optimal compressive stiffness depends on body weight and the loading profile placed on the device and thus is unique to each individual. An amputee is preparing to run in the Olympic qualifiers. You are a mechanical engineering consultant for his team and have characterized the stiffness (in kN/m) of five off-the-shelf prostheses at different stiffness categories using the Instron to the right. You have instructed him to run on each prosthesis to understand the effect of mechanical stiffness on his running speed. His average running speed in the 800 m race with each of the off-the-shelf prostheses is reported in the table below. Your goal is to determine the optimal spring stiffness to design a custom device for the Olympic athlete to maximize his performance in the 800 m race.

You have compiled the test data as tabulated below:

Test Number	k (kN/m)	v (m/s)
1	16.2	8.3
2	21.0	9.8
3	24.6	10.2
4	27.9	9.6
5	32.5	7.9

(a) What type of numerical method would you like to use if you want to discover a relation of k to v?

(b) According to the table above, describe the equation or algorithm you would use to predict v, given k. Describe all necessary steps. However, it is not necessary to perform the actual computation.

(c) Based on the equation developed in (b), how would you determine the optimal spring stiffness to maximize running speed?