## COMPUTER-AIDED ENGINEERING

 Ph.D. QUALIFIER EXAM - SPRING 2019THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG. GEORGIA INSTITUTE OF TECHNOLOGY

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- All questions in this exam have a common theme: Automobiles
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.


## Question 1 - Geometric Modeling

As a product engineer, you are given a task to design a windshield glass, as shown in the picture. You are asked to use Bézier surface patches to model.

You are given the design of Patch 1 with the control points as


Patch 1:

| (272) | (473) | (1275) |
| :---: | :---: | :---: |
| (2 4 2) | (4 4 4) | (1245) |
| (2 1 1) | (4 1 3) | (12 14 ) |

For Patch 2, some of the control points are known, and some points need to be determined.

Patch 2:

| $\mathbf{P}_{\mathbf{1}}$ | $\mathbf{P}_{\mathbf{3}}$ | $(2172)$ |
| :--- | :--- | :--- |
| $\mathbf{P}_{\mathbf{2}}$ | $\mathbf{P}_{\mathbf{4}}$ | $(2142)$ |
| $(1214)$ | $\mathbf{P}_{\mathbf{5}}$ | $(2111)$ |

a) To ensure the $\mathrm{C}^{1}$ continuity between the two patches, what are the coordinate values of $\mathbf{P}_{1}, \mathbf{P}_{2}, \mathbf{P}_{3}, \mathbf{P}_{4}$, and $\mathbf{P}_{5}$ ? Explain in detail how you determined the values showing your calculations.
b) Derive the equation of the Bézier surface patch in a matrix form.
c) Calculate the unit normal vector of Patch 1 at control point $(\mathbf{2 , 7 , 2})$.
d) If control point $(4,7,3)$ in Patch 1 is moved to a new position $(4,7,5)$, how will the new position affect the shapes of the two surface patches?

## 2) Finite-Element Analysis

A rough schematic of a bracket structure from an automobile is shown in the picture.
Select a suitable finite element to mesh the geometry, assuming the structure to be "thin" in the out-of-plane direction. You are asked to determine the stress/strain distribution throughout the structure.

1) Sketch the mesh and boundary conditions for the following cases:
a. Material is isotropic. The load

$P$ is applied vertically
downwards as shown.
b. Material is orthotropic. The load $P$ is applied vertically downwards.
c. Material is isotropic. The load is $P$ is applied horizontally.
2) Assume now that the material isotropic and the load $P$ is applied vertically downwards as shown. You are asked to determine the downward displacement. Assuming that the structure can be simplified as a trapezoidal structure with a width of W1 at the top and W2 (equal to $2 \times \mathrm{R} 2$ ) at the bottom, determine the downward extension of the structure. Assume that the length or height of the structure is $L$ and the thickness is $t$. Your computations do not need to include the hole at the bottom. Show your steps. You need not do the computation. Will your element provide an accurate determination of the downward displacement? Explain.

## Element Stiffness Matrix

$[K]=\frac{E A}{L}\left[\begin{array}{cccc}l^{2} & l m & -l^{2} & -l m \\ l m & m^{2} & -l m & -m^{2} \\ -l^{2} & -l m & l^{2} & l m \\ -l m & -m^{2} & l m & m^{2}\end{array}\right]$
where $E, A$, and $L$ are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l$ and $m$ are direction cosines.

## Question 3: Numerical Methods

A car manufacturer believes they have created a more efficient engine for handling both hilly and flat terrains. They tested their energy consumption (reported as gallons of gasoline burned per hour) in a range of different conditions. The first condition was low speed but going uphill. The second was low speed but flat terrain. The third was high speed on flat terrain.
 The fourth was high speed on hilly terrain. The tester recorded the average speed (in mph ) during an hour of driving as well as the average torque produced (in Nm ).

The rate of gasoline consumption rate $p$ in terms of gallons expended per hour to depended on the speed of the car ( v in mph ) and the torque produced by the car ( T ).

An investigator reported the data as tabulated below:

| Observations | $p$ (gal/hour) | $\mathrm{v}(\mathrm{mph})$ | $\mathrm{T}(\mathrm{Nm})$ |
| :---: | :---: | :---: | :---: |
| 1 | 1.77 | 30 | 70 |
| 2 | 1.25 | 40 | 30 |
| 3 | 2.01 | 60 | 40 |
| 4 | 3.2 | 50 | 80 |

(a) What type of numerical method would you like to use if you want to determine a relation of $p$ to $v$ and $T$ ?
(b) According to the table above, describe the equation or algorithm you would use to predict $p$, given $v$ and $T$. Describe all necessary steps. However, it is not necessary to perform the actual computation.
(c) If you were to reformulate the problem to only include a single independent variable (instead of 2), what variable will be a good choice and why?
(d) If you had data from a competitor's car tested under the same conditions, what would the hypothesis test be to definitively show that your car is more efficient than the competitor's car in all conditions?

