COMPUTER-AIDED ENGINEERING
Ph.D. QUALIFIER EXAM – FALL 2019

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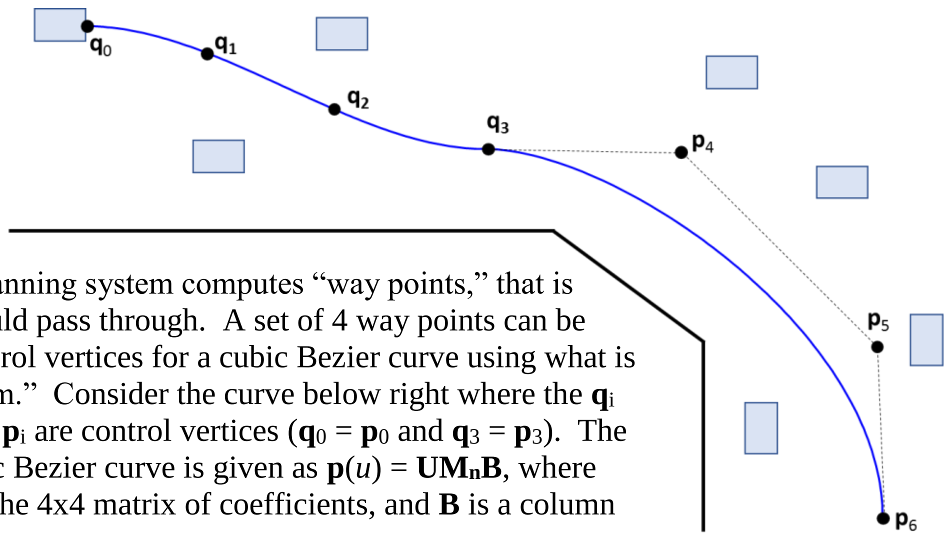
- All questions in this exam have a common theme: ***Self-Driving Cars***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

Self-driving cars need to sense their environments and update their planned trajectories so that other cars and obstacles can be avoided, while ensuring a smooth, comfortable ride. This problem will address issues of trajectory planning by investigating curve models of driving paths. A driving situation is illustrated below, with a car on the left and other cars and obstacles indicated by boxes. A proposed path is provided that is modeled by 2 cubic Bezier curves with C1 continuity.



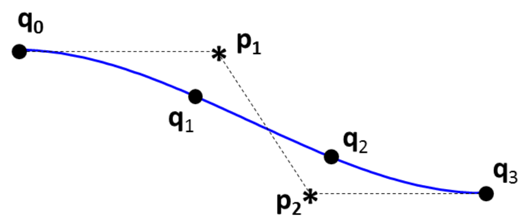
The car's trajectory planning system computes "way points," that is points that the car should pass through. A set of 4 way points can be converted into the control vertices for a cubic Bezier curve using what is called the "4-point form." Consider the curve below right where the $\mathbf{q}_i$ are way points and the $\mathbf{p}_i$ are control vertices ($\mathbf{q}_0 = \mathbf{p}_0$ and $\mathbf{q}_3 = \mathbf{p}_3$). The matrix form for a cubic Bezier curve is given as $\mathbf{p}(u) = \mathbf{U}\mathbf{M}_n\mathbf{B}$, where $\mathbf{U} = [u^3\ u^2\ u\ 1]$, $\mathbf{M}_n$ is the 4x4 matrix of coefficients, and $\mathbf{B}$ is a column vector of control vertices.

The 4-point form can be formulated as:

Given $\mathbf{q}_0$, $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$,

Find $[\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$.

Assume that $\mathbf{q}_1$ is given at $u = 1/3$, and $\mathbf{q}_2$ is at $u = 2/3$.



The control vertex positions are: $\mathbf{q}_0 = (0, 50)$, $\mathbf{q}_1 = (15, 47.4)$, $\mathbf{q}_2 = (30, 42.6)$, $\mathbf{q}_3 = (45, 40)$, $\mathbf{p}_4 = (?, ?)$, $\mathbf{p}_5 = (90, 25)$, $\mathbf{p}_6 = (90, 10)$.

$$\boldsymbol{p}(u) = \sum_{i=0}^{n} \mathbf{p}_i B_{i,n}(u)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Answer the following questions:

a) Outline a procedure for computing $\mathbf{p}_1$ and $\mathbf{p}_2$, the unknown control vertex positions, for the 4-point form.

b) Assume that after solving part a) the equation for $\mathbf{p}_1$ and $\mathbf{p}_2$ is given as:

$$\begin{bmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \end{bmatrix} = \begin{bmatrix} -0.8333 & 3 & -1.5 & 0.3333 \\ 0.3333 & -1.5 & 3 & -0.8333 \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_0 \\ \boldsymbol{q}_1 \\ \boldsymbol{q}_2 \\ \boldsymbol{q}_3 \end{bmatrix}$$

Compute $\mathbf{p}_1$ and $\mathbf{p}_2$.

c) Compute the point on the curve at $u = 0.5$.

d) Compute the coordinates of $\mathbf{p}_4$ to ensure C1 continuity.

2) Finite-Element Analysis

Many companies start to test self-driving cars. Typically, a camera is mounted on top of a car for testing. The stability of the camera is critical to the safety as well as to the success of the tests. You are asked to analyze the design of the supporting frame using finite-element formulation (see the figure below). The vertical force due to the camera weight is F_2=200N. The horizontal force due to car acceleration/braking is estimated F_1=40N. The cross-sectional area of each truss is 25mm^2. The Young's modulus of the material is E=100GPa.

2.1 Consider the design below to find the displacements at node 1.

- Use the minimum number of elements.
- State your assumptions clearly.
- Write down the element stiffness matrices.
- Assemble element stiffness matrices into a global stiffness matrix.
- Introduce the boundary conditions to be able to solve the problem.
- You do not need to solve the problem.

2.2 The design shown in the figures appears to be heavy. To reduce the weight, one suggestion is to remove truss 4. Could you please comment on how this change in the design will affect the finite-element analysis?

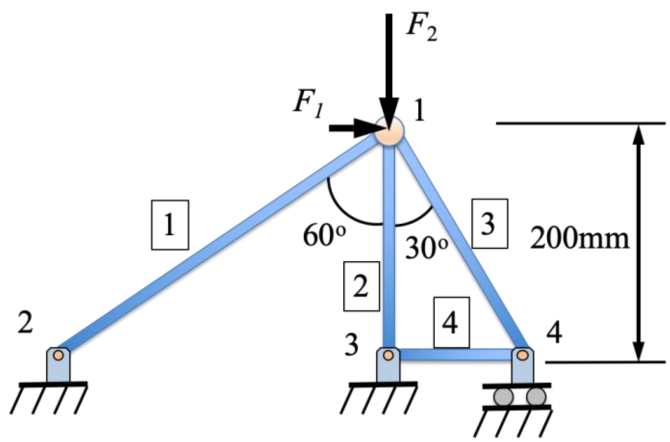


Figure for problem 2. Design of the structure for holding the camera

Element Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where *E, A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines.

3) Numerical Analysis

You are developing a traffic flow model for finding the optimal velocity of self-driving vehicles. Assume that the movement of a car can be approximately described with the following equation:

$$\frac{d^2x(t)}{dt^2} = s\left(F_m(t) - K_f\frac{dx(t)}{dt} - K_a\left(\frac{dx(t)}{dt}\right)^2\right) \qquad \textbf{(1)}$$

where x(t) is the position of the car at time t, s denotes a sensitivity parameter which describes how sensitive the average driver is to the motion of the car in front of him/her. F_m, K_f, and K_a are the tractive force from the motor, viscous friction coefficient, and aerodynamic drag coefficient, respectively.

Consider the case of $s = 0.5, F_m(\mathrm{t}) = 11e^{-t} - 5x(\mathrm{t})$, K_f = 3 N s/m, and K_a = 0 N s^2/m^2.

(a) Write down a state space representation (first-order system equations) for x(t) to describe the position of the car.

(b) Conduct the solution process of the given equation using Heun's method by assuming $x(0)=7$, and $x'(0)=13$ with a step size of 0.25. Compute one iteration only.

(c) If an extremely small value is used for the step size, how would you expect the solution to change? What are the potential issues you can think of? How can you avoid these issues without changing the step size already used? Provide a detailed explanation.

(d) Now, consider **Eq. (1)** again, but assume the aerodynamic drag coefficient, K_a, is not negligible. How can you handle this problem? What are the potential issues or benefits of performing your suggested method?

The equations for Heun's method are:

$$\begin{cases} y_{i+1} = y_i + h\phi \\ \phi = (k_1 + k_2)/2 \\ k_1 = f(t_i, y_i) \\ k_2 = f(t_i + h, y_i + hk_1) \end{cases}$$