Georgia Institute of Technology

The George W. Woodruff School of Mechanical Engineering
Nuclear & Radiological Engineering/Medical Physics Program

Ph.D. Qualifier Exam

Fall Semester 2009

_______________ Your ID Code

Radiation Physics (Day 1)

Instructions

1. Use a separate page for each answer sheet (no front to back answers).
2. The question number should be shown on each answer sheet.
3. ANSWER 4 OF 6 QUESTIONS ONLY.
4. Staple your question sheet to your answer sheets and turn in.

NRE/MP Radiation Physics

<u>Answer any 4 of the following 6 questions</u>.

1. In an alloyed Am(Be) neutron source, neutrons are produced from the interactions of 5.5-MeV alpha particles (emitted from ^{241}Am) with the ^{9}Be nuclei. That is,

$$ {}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{1}_{0}n + {}^{12}_{6}C $$

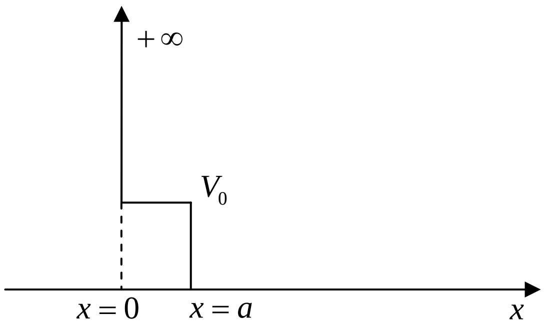
   a. Use the mass table (attachment A) to calculate the kinetic energy of the alpha particle.
   b. Given that the nuclear radius obeys the formula, $R = 1.25 \times A^{1/3}\ \ fm$ and that $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44$ *MeV fm*, use the classical approach to estimate the coulomb barrier (in *MeV*) for the above (α,n) reaction.
   c. Use the classical approach to estimate the cross section (in barns) for the above (α,n) reaction, and discuss how the cross section should be modified by the quantum-mechanical approach.

2. As a follow-up question of problem 1, use the mass table (attachment C) to calculate the energy range of neutrons emitted in the LAB system.

3. Show that it is impossible for a gamma photon to conserve momentum in pair production without the presence of a third body (i.e. a nucleus or an electron).

4. For the following 1D potential function,

$$ V(x) = \begin{cases} +\infty, x < 0 \\ V_0, 0 \le x \le a \\ 0, x > a \end{cases} $$



where V_0 is positive.

   a. Assuming particles are incident from $x = +\infty$ in the direction toward $-\infty$, with energies $0 < E < V_0$, write down the Schrodinger equations for different regions.
   b. Using boundary conditions, solve the Schrodinger equation. Evaluate all undetermined coefficients in terms of a single common coefficient, but do not attempt to normalize the wave function.
   c. In the region $x > a$, based on the solution of b), calculate the wave reflection coefficient.

5. The ordering of the single particle nuclear energy levels is

$$1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 2s_{1/2}, 1d_{3/2}, 1f_{7/2}, 2p_{3/2} \ldots$$

For the following nuclei ${}^{7}_{3}Li, {}^{23}_{11}Na, {}^{33}_{16}S, {}^{41}_{21}Sc$,

a. Using the independent-particle shell model, find the configurations of the protons and neutrons, and then determine the ground state spin and parity.

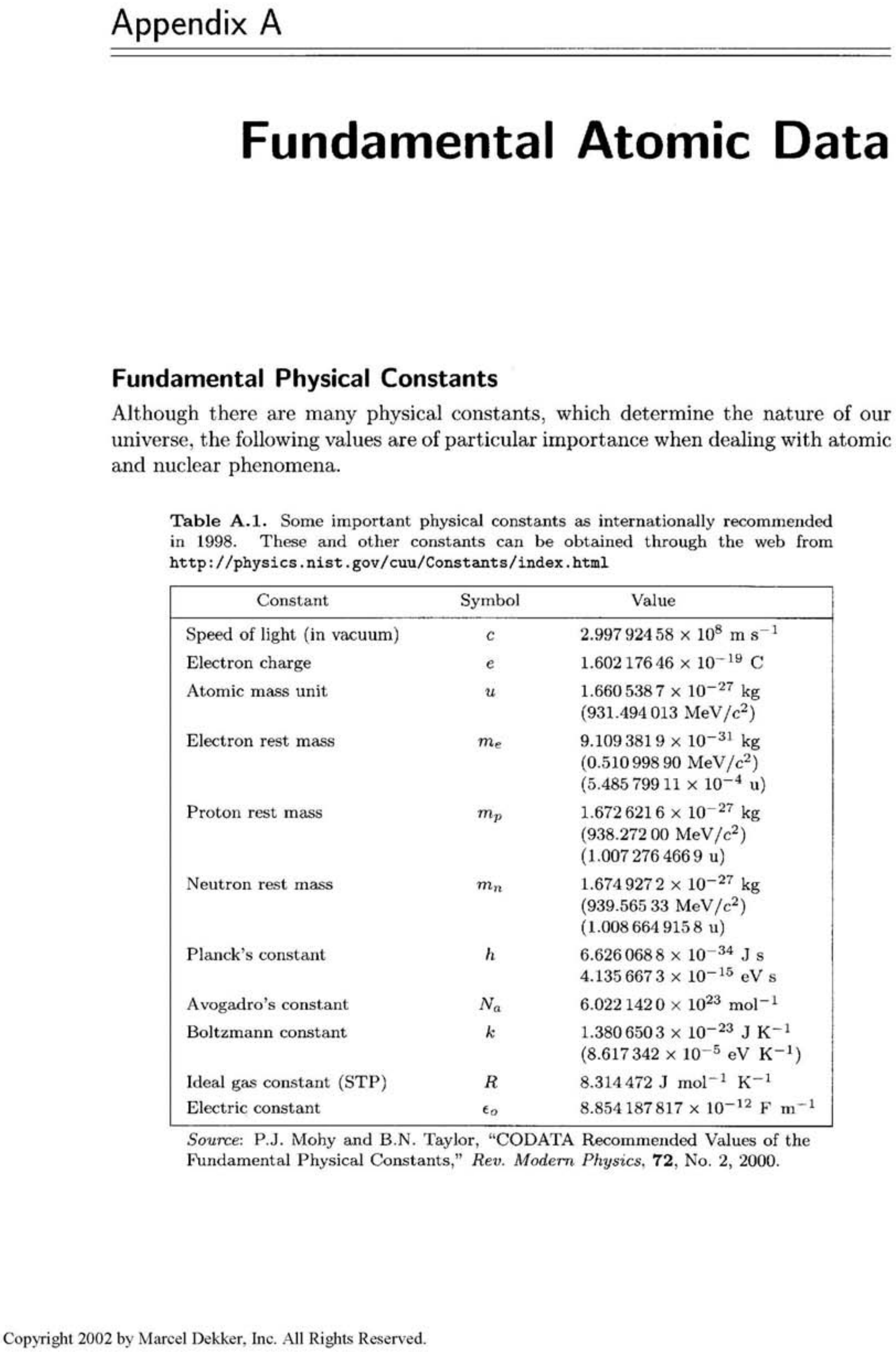
b. In the shell model, the first excited states can be produced either by

    1. excitation of the unpaired nucleon into the next higher subshell, or
    2. pairing this nucleon with another excited from the next lower subshell.

Determine the spin and parity for these two types of excited states for each of the four given nuclides.

6. Consider the 3 component decay chain $X_1 \xrightarrow{\lambda_1} X_2 \xrightarrow{\lambda_2} X_3$.

    a. Derive an expression for the amount and the activities of X_2 and X_3 at any time t as a function of decay constants. Assume at time t=0, only N_1^0 amount of X1 is present.
    b. When will these radionuclides be in secular equilibrium?



Appendix A

Fundamental Atomic Data

Fundamental Physical Constants

Although there are many physical constants, which determine the nature of our universe, the following values are of particular importance when dealing with atomic and nuclear phenomena.

Table A.1. Some important physical constants as internationally recommended in 1998. These and other constants can be obtained through the web from `http://physics.nist.gov/cuu/Constants/index.html`

Constant	Symbol	Value
Speed of light (in vacuum)	c	$2.997\,924\,58 \times 10^{8}$ m s^{-1}
Electron charge	e	$1.602\,176\,46 \times 10^{-19}$ C
Atomic mass unit	u	$1.660\,538\,7 \times 10^{-27}$ kg (931.494 013 MeV/c^2)
Electron rest mass	m_e	$9.109\,381\,9 \times 10^{-31}$ kg (0.510 998 90 MeV/c^2) ($5.485\,799\,11 \times 10^{-4}$ u)
Proton rest mass	m_p	$1.672\,621\,6 \times 10^{-27}$ kg (938.272 00 MeV/c^2) (1.007 276 466 9 u)
Neutron rest mass	m_n	$1.674\,927\,2 \times 10^{-27}$ kg (939.565 33 MeV/c^2) (1.008 664 915 8 u)
Planck's constant	h	$6.626\,068\,8 \times 10^{-34}$ J s $4.135\,667\,3 \times 10^{-15}$ eV s
Avogadro's constant	N_a	$6.022\,142\,0 \times 10^{23}$ mol^{-1}
Boltzmann constant	k	$1.380\,650\,3 \times 10^{-23}$ J K^{-1} ($8.617\,342 \times 10^{-5}$ eV K^{-1})
Ideal gas constant (STP)	R	8.314 472 J mol^{-1} K^{-1}
Electric constant	ϵ_o	$8.854\,187\,817 \times 10^{-12}$ F m^{-1}

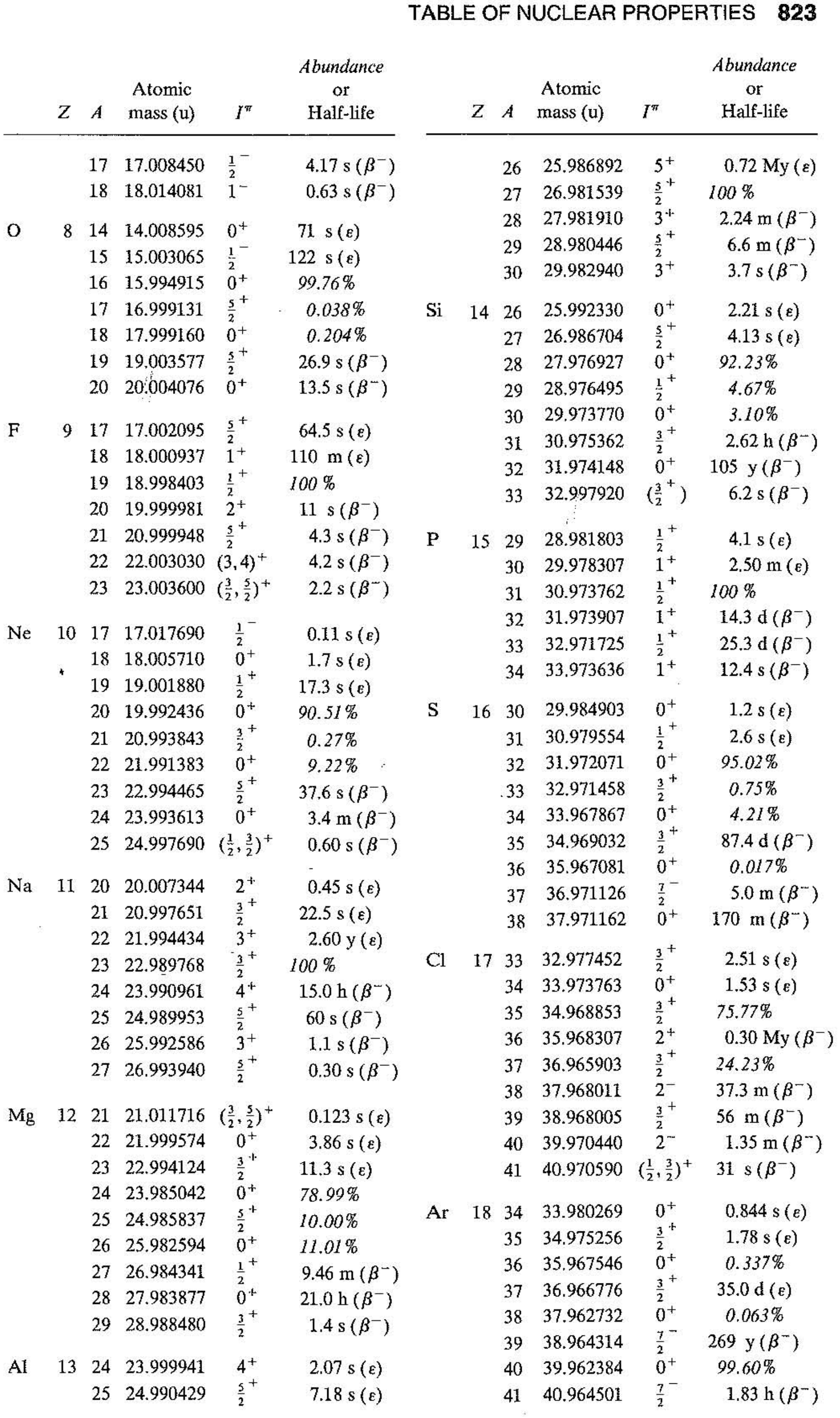
Source: P.J. Mohy and B.N. Taylor, "CODATA Recommended Values of the Fundamental Physical Constants," *Rev. Modern Physics*, **72**, No. 2, 2000.

Appendix C

TABLE OF NUCLEAR PROPERTIES

The following table shows some properties of a selection of isotopes. For each element only the stable and relatively long-lived radioactive isotopes are included. Ground-state atomic masses and spin-parity assignments are shown for all isotopes; uncertain spin-parity assignments are in parentheses. Abundances are given for stable isotopes, and for radioactive isotopes the half-life and principal decay mode are shown (ε—electron capture, possibly including positron emission; β^-—negative beta decay; α—alpha decay; f—spontaneous fission). The masses are those of the corresponding neutral atoms and were taken from the 1983 atomic mass evaluation: A. H. Wapstra and G. Audi, *Nucl. Phys.* **A432**, 1 (1985). In the half-life entries, My = 10^6 y. Uncertainties in the masses are typically 10^{-5} u (10^{-4} u for some cases far from stability); uncertainties in the abundances and half-lives are typically at or below the level of the last digit tabulated.

	Z	A	Atomic mass (u)	I^π	*Abundance* or Half-life
H	1	1	1.007825	$\frac{1}{2}^+$	*99.985%*
		2	2.014102	1^+	*0.015%*
		3	3.016049	$\frac{1}{2}^+$	12.3 y (β^-)
He	2	3	3.016029	$\frac{1}{2}^+$	$1.38 \times 10^{-4}\%$
		4	4.002603	0^+	*99.99986%*
Li	3	6	6.015121	1^+	*7.5%*
		7	7.016003	$\frac{3}{2}^-$	*92.5%*
		8	8.022486	2^+	0.84 s (β^-)
Be	4	7	7.016928	$\frac{3}{2}^-$	53.3 d (ε)
		8	8.005305	0^+	0.07 fs (α)
		9	9.012182	$\frac{3}{2}^-$	*100 %*
		10	10.013534	0^+	1.6 My (β^-)
		11	11.021658	$\frac{1}{2}^+$	13.8 s (β^-)
B	5	8	8.024606	2^+	0.77 s (ε)
		9	9.013329	$\frac{3}{2}^-$	0.85 as (α)
		10	10.012937	3^+	*19.8%*
		11	11.009305	$\frac{3}{2}^-$	*80.2%*
		12	12.014353	1^+	20.4 ms (β^-)
		13	13.017780	$\frac{3}{2}^-$	17.4 ms (β^-)
C	6	9	9.031039	$\frac{3}{2}^-$	0.13 s (ε)
		10	10.016856	0^+	19.2 s (ε)
		11	11.011433	$\frac{3}{2}^-$	20.4 m (ε)
		12	12.000000	0^+	*98.89%*
		13	13.003355	$\frac{1}{2}^-$	*1.11%*
		14	14.003242	0^+	5730 y (β^-)
		15	15.010599	$\frac{1}{2}^+$	2.45 s (β^-)
N	7	12	12.018613	1^+	11 ms (ε)
		13	13.005739	$\frac{1}{2}^-$	9.96 m (ε)
		14	14.003074	1^+	*99.63%*
		15	15.000109	$\frac{1}{2}^-$	*0.366%*
		16	16.006100	2^-	7.13 s (β^-)



	Z	A	Atomic mass (u)	I^π	*Abundance* or Half-life
		17	17.008450	$\frac{1}{2}^-$	4.17 s (β^-)
		18	18.014081	1^-	0.63 s (β^-)
O	8	14	14.008595	0^+	71 s (ε)
		15	15.003065	$\frac{1}{2}^-$	122 s (ε)
		16	15.994915	0^+	*99.76%*
		17	16.999131	$\frac{5}{2}^+$	*0.038%*
		18	17.999160	0^+	*0.204%*
		19	19.003577	$\frac{5}{2}^+$	26.9 s (β^-)
		20	20.004076	0^+	13.5 s (β^-)
F	9	17	17.002095	$\frac{5}{2}^+$	64.5 s (ε)
		18	18.000937	1^+	110 m (ε)
		19	18.998403	$\frac{1}{2}^+$	*100 %*
		20	19.999981	2^+	11 s (β^-)
		21	20.999948	$\frac{5}{2}^+$	4.3 s (β^-)
		22	22.003030	$(3,4)^+$	4.2 s (β^-)
		23	23.003600	$(\frac{3}{2},\frac{5}{2})^+$	2.2 s (β^-)
Ne	10	17	17.017690	$\frac{1}{2}^-$	0.11 s (ε)
		18	18.005710	0^+	1.7 s (ε)
		19	19.001880	$\frac{1}{2}^+$	17.3 s (ε)
		20	19.992436	0^+	*90.51%*
		21	20.993843	$\frac{3}{2}^+$	*0.27%*
		22	21.991383	0^+	*9.22%*
		23	22.994465	$\frac{5}{2}^+$	37.6 s (β^-)
		24	23.993613	0^+	3.4 m (β^-)
		25	24.997690	$(\frac{1}{2},\frac{3}{2})^+$	0.60 s (β^-)
Na	11	20	20.007344	2^+	0.45 s (ε)
		21	20.997651	$\frac{3}{2}^+$	22.5 s (ε)
		22	21.994434	3^+	2.60 y (ε)
		23	22.989768	$\frac{3}{2}^+$	*100 %*
		24	23.990961	4^+	15.0 h (β^-)
		25	24.989953	$\frac{5}{2}^+$	60 s (β^-)
		26	25.992586	3^+	1.1 s (β^-)
		27	26.993940	$\frac{5}{2}^+$	0.30 s (β^-)
Mg	12	21	21.011716	$(\frac{3}{2},\frac{5}{2})^+$	0.123 s (ε)
		22	21.999574	0^+	3.86 s (ε)
		23	22.994124	$\frac{3}{2}^+$	11.3 s (ε)
		24	23.985042	0^+	*78.99%*
		25	24.985837	$\frac{5}{2}^+$	*10.00%*
		26	25.982594	0^+	*11.01%*
		27	26.984341	$\frac{1}{2}^+$	9.46 m (β^-)
		28	27.983877	0^+	21.0 h (β^-)
		29	28.988480	$\frac{3}{2}^+$	1.4 s (β^-)
Al	13	24	23.999941	4^+	2.07 s (ε)
		25	24.990429	$\frac{5}{2}^+$	7.18 s (ε)
		26	25.986892	5^+	0.72 My (ε)
		27	26.981539	$\frac{5}{2}^+$	*100 %*
		28	27.981910	3^+	2.24 m (β^-)
		29	28.980446	$\frac{5}{2}^+$	6.6 m (β^-)
		30	29.982940	3^+	3.7 s (β^-)
Si	14	26	25.992330	0^+	2.21 s (ε)
		27	26.986704	$\frac{5}{2}^+$	4.13 s (ε)
		28	27.976927	0^+	*92.23%*
		29	28.976495	$\frac{1}{2}^+$	*4.67%*
		30	29.973770	0^+	*3.10%*
		31	30.975362	$\frac{3}{2}^+$	2.62 h (β^-)
		32	31.974148	0^+	105 y (β^-)
		33	32.997920	$(\frac{3}{2}^+)$	6.2 s (β^-)
P	15	29	28.981803	$\frac{1}{2}^+$	4.1 s (ε)
		30	29.978307	1^+	2.50 m (ε)
		31	30.973762	$\frac{1}{2}^+$	*100 %*
		32	31.973907	1^+	14.3 d (β^-)
		33	32.971725	$\frac{1}{2}^+$	25.3 d (β^-)
		34	33.973636	1^+	12.4 s (β^-)
S	16	30	29.984903	0^+	1.2 s (ε)
		31	30.979554	$\frac{1}{2}^+$	2.6 s (ε)
		32	31.972071	0^+	*95.02%*
		33	32.971458	$\frac{3}{2}^+$	*0.75%*
		34	33.967867	0^+	*4.21%*
		35	34.969032	$\frac{3}{2}^+$	87.4 d (β^-)
		36	35.967081	0^+	*0.017%*
		37	36.971126	$\frac{7}{2}^-$	5.0 m (β^-)
		38	37.971162	0^+	170 m (β^-)
Cl	17	33	32.977452	$\frac{3}{2}^+$	2.51 s (ε)
		34	33.973763	0^+	1.53 s (ε)
		35	34.968853	$\frac{3}{2}^+$	*75.77%*
		36	35.968307	2^+	0.30 My (β^-)
		37	36.965903	$\frac{3}{2}^+$	*24.23%*
		38	37.968011	2^-	37.3 m (β^-)
		39	38.968005	$\frac{3}{2}^+$	56 m (β^-)
		40	39.970440	2^-	1.35 m (β^-)
		41	40.970590	$(\frac{1}{2},\frac{3}{2})^+$	31 s (β^-)
Ar	18	34	33.980269	0^+	0.844 s (ε)
		35	34.975256	$\frac{3}{2}^+$	1.78 s (ε)
		36	35.967546	0^+	*0.337%*
		37	36.966776	$\frac{3}{2}^+$	35.0 d (ε)
		38	37.962732	0^+	*0.063%*
		39	38.964314	$\frac{7}{2}^-$	269 y (β^-)
		40	39.962384	0^+	*99.60%*
		41	40.964501	$\frac{7}{2}^-$	1.83 h (β^-)