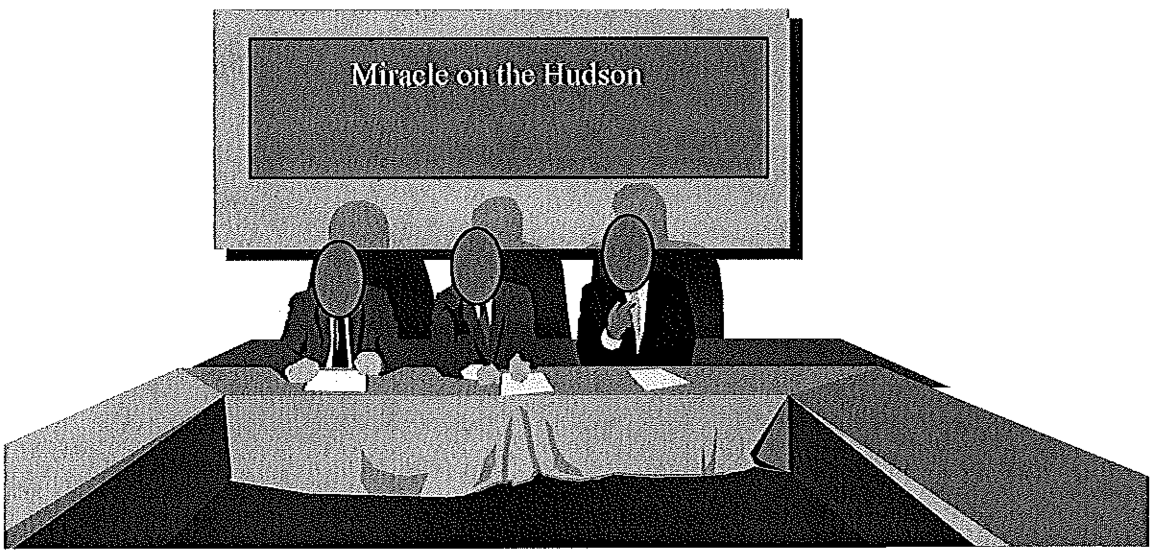
COMPUTER-AIDED ENGINEERING
Ph.D. QUALIFIER EXAM – Spring 2009

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

S.-K. Choi, C. Paredis, D. Rosen, and S. Sitaraman (Chair)



- All questions in this exam have a common theme: ***Miracle on the Hudson***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

Recently, US Airways flight 1549 landed in the Hudson River near New York City. Assume that its trajectory was given by a quadratic Bezier curve **p**(u). You are to determine the location where the airplane hits the water by intersecting **p**(u) with the river, **q**(w). See Figure 1.

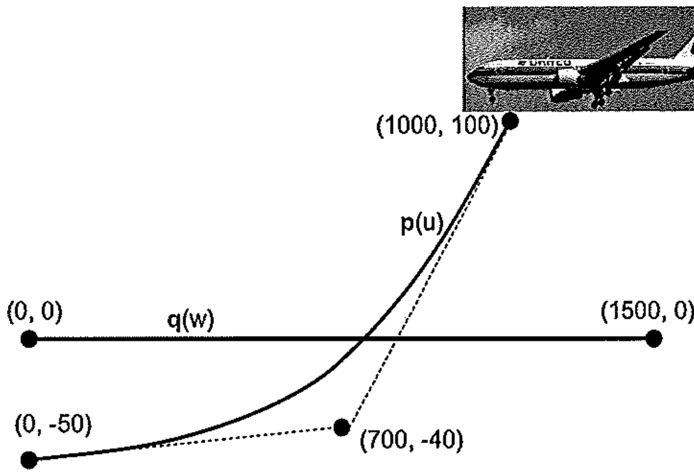


Figure 1

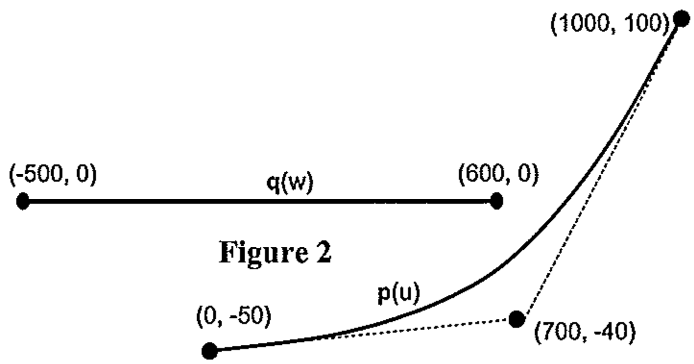


Figure 2

As a reminder, the equation for a Bezier curve is:

$$\mathbf{q}(u) = \sum_{i=0}^{n} \mathbf{q}_i B_{i,n}(u)$$

where the $B_{i,n}(u)$, etc., are the Bezier blending functions.

a. Describe the condition mathematically for the path of the plane to intersect the river. Describe this condition verbally as well.

b. Derive the equation for the plane's path (not a function of time). Derive the equation for the river.

c. Derive the equations for the intersection condition.

d. Solve the equations for the coordinates of the point where the plane hits the water. Identify the numerical or analytical method that you used.

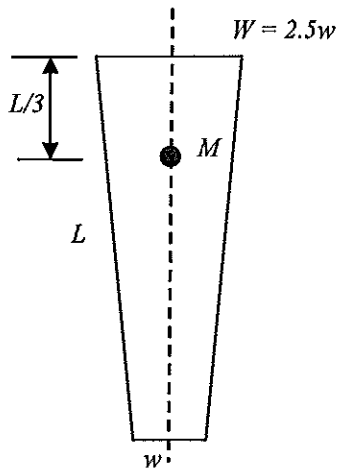
e. Describe what happens mathematically in your solution method if the curves do not actually intersect, as illustrated in Figure 2.

f. Identify another numerical or analytical method that you could use to solve either this problem or a more general problem of intersecting Bezier curves.

Question 2 – Finite Element Analysis

The photo below shows the passengers standing the wings of the plane. To get initial estimates of wing tip deflection, an engineer models the wing as a simple trapezoid as shown in the sketch below. As illustrated, the wing has a base width W, tip width w, and length L. The base width $W = 2.5w$. Assume that the wing is attached to the fuselage where the width is W. Assume that the wing has a rectangular cross-section with a uniform thickness T.

Assume that the engine with a mass of M is attached to the wing at 1/3rd L from the fuselage. Also, assume that the passengers are of equal mass m and 24 persons are standing in one line along the center of the wing. This line is illustrated as a dashed line in the sketch.



Using appropriate finite-element formulation,

- Determine the tip deflection of the wing, assuming that the fuselage stands rigid and does not rotate. Neglect the effect of water for your calculations.
- Assume that the water can be modeled as a series of springs under the wing along the dashed line. Now, determine the tip deflection taking into consideration the resistance from water. For this computation, outline the steps. You do not need to solve.

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element with respect

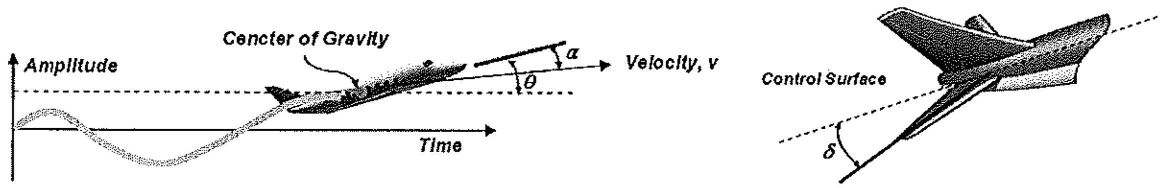
Element B - Stiffness Matrix

$$[K] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where E, I, and h are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Methods

A measure of the degree of instability in an unstable aircraft response is the amount of time it takes for the amplitude of the time response to double, given some nonzero initial condition. The equations describing the dynamics of US Airways flight 1549 can be obtained by using Newton's laws and the angles defined in the figure below.



To simplify the equations, we will assume that θ is a small angle and the velocity v is constant and equal to 25ft/s. The state variables of the aircraft, considering only vertical control, are $x_1 = \theta$, $x_2 = d\theta/dt$, and $x_3 = \alpha$, where α is the angle of attack. Thus, the state vector differential equation for this system is given by

$$\dot{x} = \mathrm{A}x + \mathrm{B}u(t) \quad \text{...} \quad \text{Eq. (1)}$$

where $u(\mathrm{t}) = \delta(\mathrm{t})$, the deflection of the tail plane.

Task 1: Find the response of the system represented by a state vector differential equation (Eq. 1), by utilizing a discrete-time approximation:

$$\dot{x}(t) = \frac{x(t+T) - x(t)}{T}$$

where t is divided into intervals of width T ($t=kT$ and $k=0,1,2,3,\ldots$).

Task 2: Using the obtained equation from Task 1, determine the responses of the system at the first two time instances, such as $k=0$ and $k=1$. Use a time increment of T equal to 0.2 seconds, $u(0)=1$ and $u(k)=0$ for $k \geq 1$. A and B are given as

$$A = \begin{bmatrix} -1 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Question 1: When the aircraft encounters unexpected loading conditions, i.e. turbulence, the response of the system may include several stiff equations (i.e., exponential functions) with magnitudes varying with time at a significantly different rate. Check whether the current approach can be applicable to the stiff equation, e^{-at}. Justify your answer.

Question 2: This recurrence operation (Task 1) is a sequential series of calculations. Suggest an alternative approach to improve the accuracy of the given problem. Comment on additional advantages and difficulties with the suggested approach compared to the current procedure (Task 1).